MECHANICS OF FLUIDS

COURSE CODE : **MET 203**

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OVERVIEW ABOUT COURSE

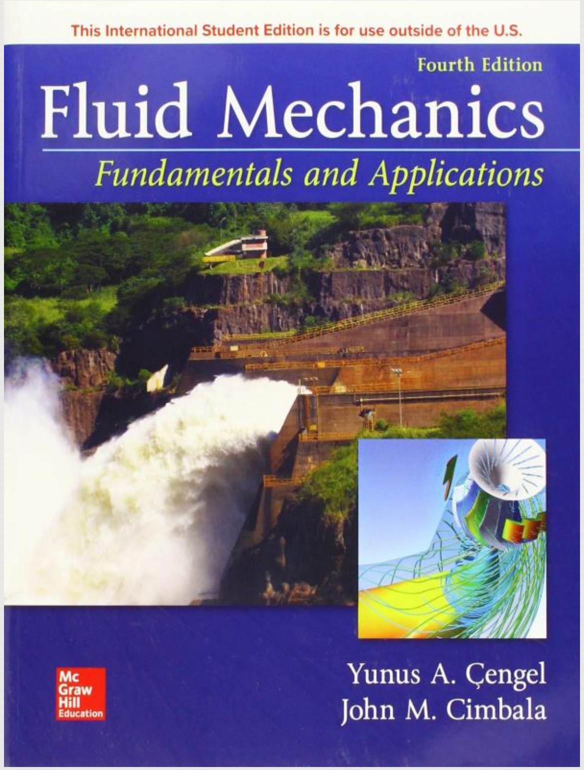
- Lecture : 5
- Tutorials :1
- Credits : 4

ETE Tentative schedule

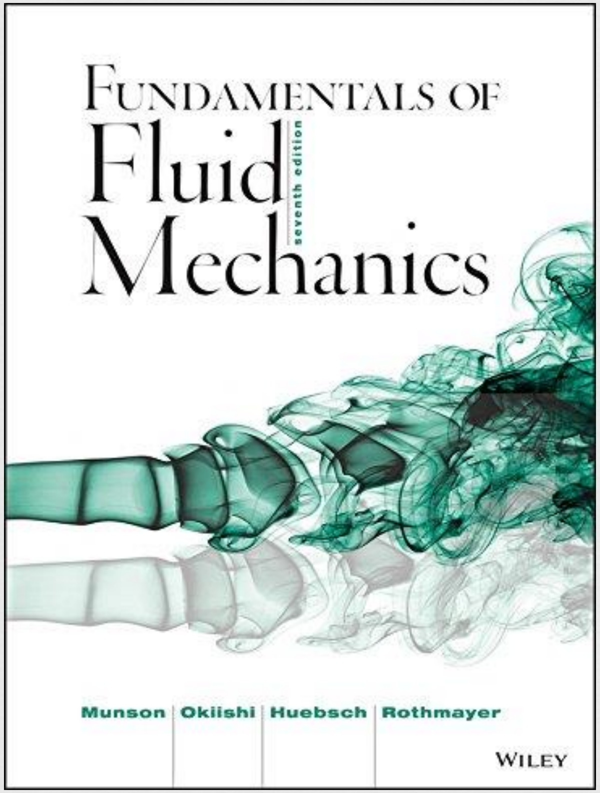
FIRST INTERNAL TEST (After 3 weeks)
SECOND INTERNAL TEST(After 7th week)
END SEMESTER EXAM

TEXT BOOKS

TEXT BOOK



REFERENCE BOOK



MECHANICS OF FLUIDS
(WHAT YOU SEE IS NOT WHAT YOU GET)

- **Course Objectives :**

This course provides an introduction to the properties and behavior of fluids. It enables to apply the concepts in engineering, pipe networks. It introduces the concepts of boundary layers, dimensional analysis and model testing

Course Outcomes

- Define Properties of Fluids and Solve hydrostatic problems
- Explain fluid kinematics and Classify fluid flows
- Interpret Euler and Navier-Stokes equations and Solve problems using Bernoulli's equation
- Evaluate energy loses in pipes and sketch energy gradient lines
- Explain the concept of boundary layer and its applications
- Use dimensional Analysis for model studies

Syllabus

- Introduction
- Kinematics of fluid flow
- Control volume analysis of mass, momentum and energy, Equations of fluid dynamics
- Pipe Flow: Viscous flow
- Boundary Layer

INTRODUCTION

Mechanics: The oldest physical science that deals with both stationary and moving bodies under the influence of forces.

Statics: The branch of mechanics that deals with bodies at rest.

Dynamics: The branch that deals with bodies in motion.

Fluid mechanics: The science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.

Fluid dynamics: Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity.

Fluid mechanics deals with liquids and gases in motion or at rest.

Hydrodynamics: The study of the motion of fluids that can be approximated as incompressible (such as liquids, especially water, and gases at low speeds).

Hydraulics: A subcategory of hydrodynamics**,** which deals with liquid flows in pipes and open channels.

Gas dynamics: Deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.

Aerodynamics: Deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.

Meteorology, oceanography, and **hydrology:** Deal with naturally occurring flows.

Application Areas of Fluid Mechanics

Fluid dynamics is used extensively in the design of artificial hearts. Shown here is the Penn State Electric Total Artificial Heart.

Natural flows and weather

Power plants

Boats

Aircraft and spacecraft

Human body

Cars

Wind turbines

Piping and plumbing systems

Industrial applications

Aerodynamics = fluid dynamics?

•Pretty much but while fluid dynamics encompass all fluids, aerodynamics focus more on air flows

•Other more specific differences:

–Aerodynamics focus on the forces acting on bodies (i.e. lift and drag forces)

–Aerodynamics typically applied to external flows (i.e. aircrafts, cars)

–Aerodynamics provide parameters for flight dynamics and control

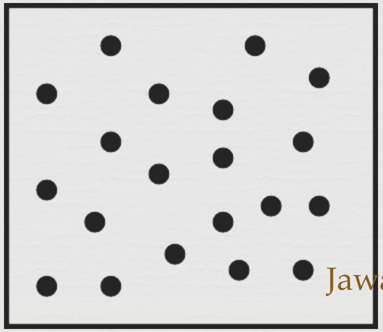
–Aerodynamics usually deal with fast moving air flows

•However, much of the governing concepts between aerodynamics and fluid dynamics are similar

Fluid as a continuum

- Air (or water, for that matter) consists of a large number of molecules
- 1 mole of air contains 6 x 10^{23} molecules (that's basically a lot!)
- Think about it – there are more molecules in a glass of water than there are glasses on Earth
- However, to understand the behaviour of air from the combined behaviour of all the molecules is simply too daunting – hence we assume air behaves like a "continuum" even if it is really not true
- **Continuum hypothesis**
  - *that fluid properties can be treated as distributed continuously in space, if one analyze them at a sufficiently large scale*

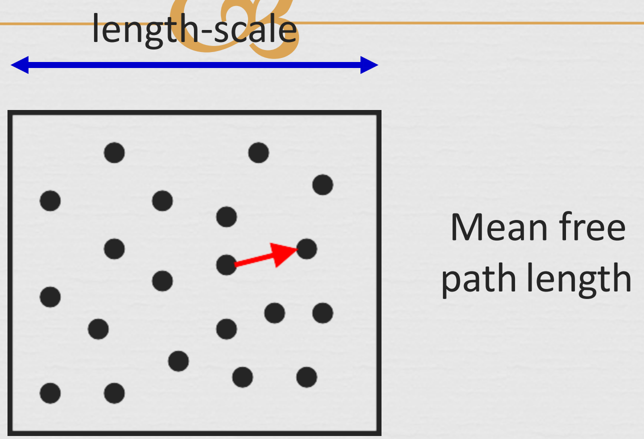
molecules



continuum

Fluid as a continuum

- In theory, so long the length-scales are larger than the mean free path length of the molecules, continuum hypothesis is valid



- This is usually almost the case for aerodynamics, except when shocks are produced
- This is because shocks are essentially discontinuities within the air flows, which violates the continuum hypothesis

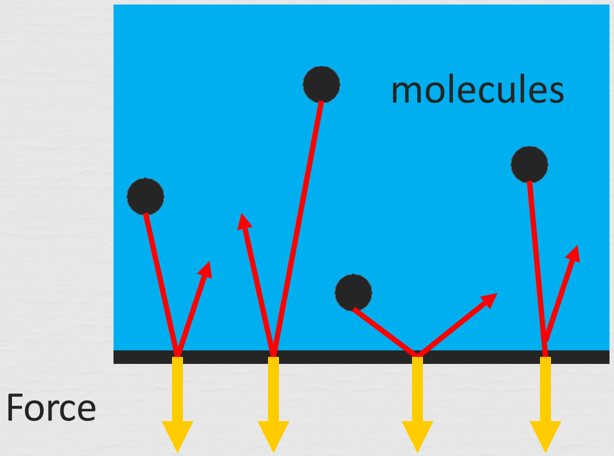
Fluid as a continuum

- But what does being a continuum imply?
  - that fluid variables (i.e. velocity, pressure, density etc) can be defined at every location within the flow field
  - that derivatives of the fluid variables (i.e. velocity gradient, pressure gradient, density gradient etc) can be determined as well

- This is despite that there are actually physical gaps (extremely small though) between the molecules

Common fluid properties

- **Temperature**
  - 288.15K at standard atmospheric conditions at sea level
- **Pressure**
  - 101300Pa (or N/m^2) at standard conditions (~10^5Pa)
  - Due to air molecules impacting upon surfaces by Brownian motion
  - Under continuum hypothesis, forces due to numerous impacts are assumed to average out over entire surface



Common fluid properties

- **Density**
  - Again due to continuum hypothesis, mass is assumed to be equally distributed within the volume
  - Quite impossible to keep track mass of each molecule (except at very small scales)

$$\text{Density}, \rho = \frac{\text{Mass}}{\text{Volume}}$$

  - Unit is clearly kg/m^3
  - Specific volume (m^3/kg), ν, is sometimes used instead of density

$$\nu = \frac{1}{\rho}$$

  - Pressure, temperature, density are related by the Ideal Gas Equation:

$$pV = mRT$$

$$\Rightarrow p = \rho RT$$

Common fluid properties

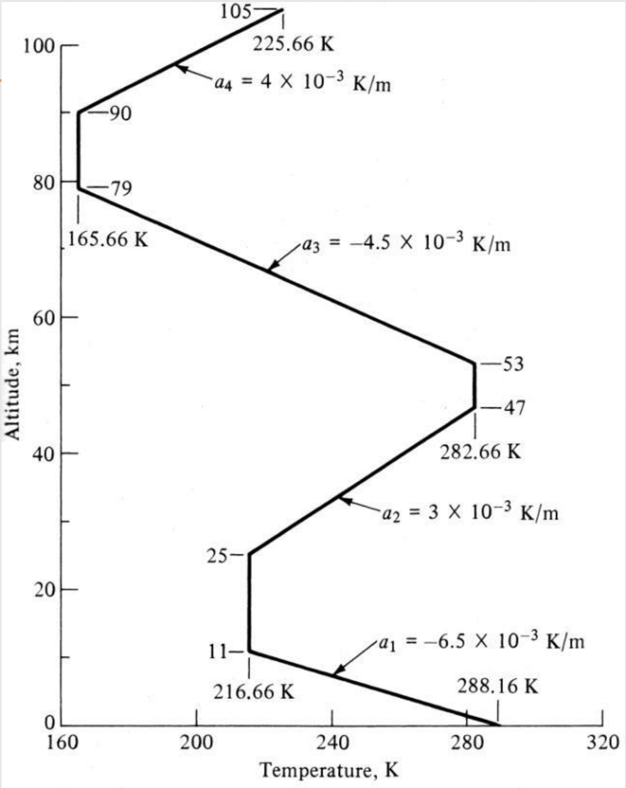
- It is straight forward to determine air density at standard conditions:

$$p = \rho R T$$

$$\Rightarrow \rho = \frac{p}{RT}$$

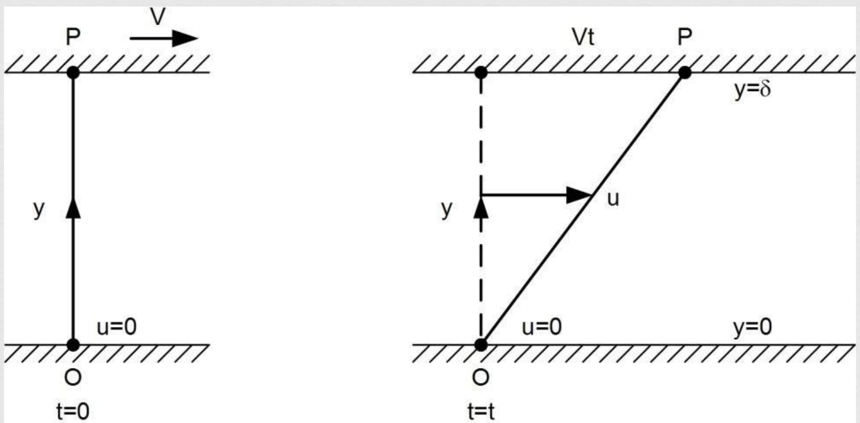
$$= \frac{101300}{(287)(288.15)}$$

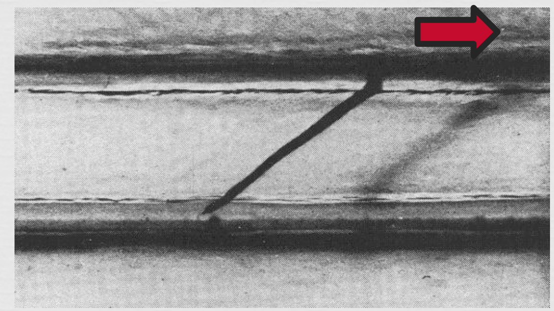
$$= 1.225 \text{kg/m}^3$$



Fluid viscosity

- In the study of fluid mechanics/aerodynamics, *fluid viscosity* is one property which plays important roles in skin friction drag, flow separation, vortex generation and many other phenomena
- How is viscosity important? Take for example a one-dimensional flow:





Actual experiment

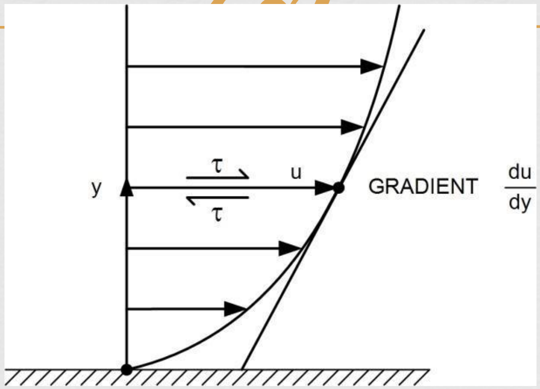
- Top plate moving but bottom plate remains stationary
- It has been established that shear stress can then be written as

$$\tau = \mu \frac{\partial u}{\partial y}$$

- *μ is the dynamic viscosity*

Fluid viscosity

- Most of the time, the velocity variation (and hence velocity gradient) is however *non-linear*



- In that case, shear stress has to calculated for each point, based on the exact velocity gradient at that point
- Clearly, shear stress will vary along y
- For two-dimensional flows, shear stress should actually be written as

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Fluid viscosity

- Viscosity is mildly sensitive to temperature changes. Semi-empirical equations are available to determine them

- For gases,

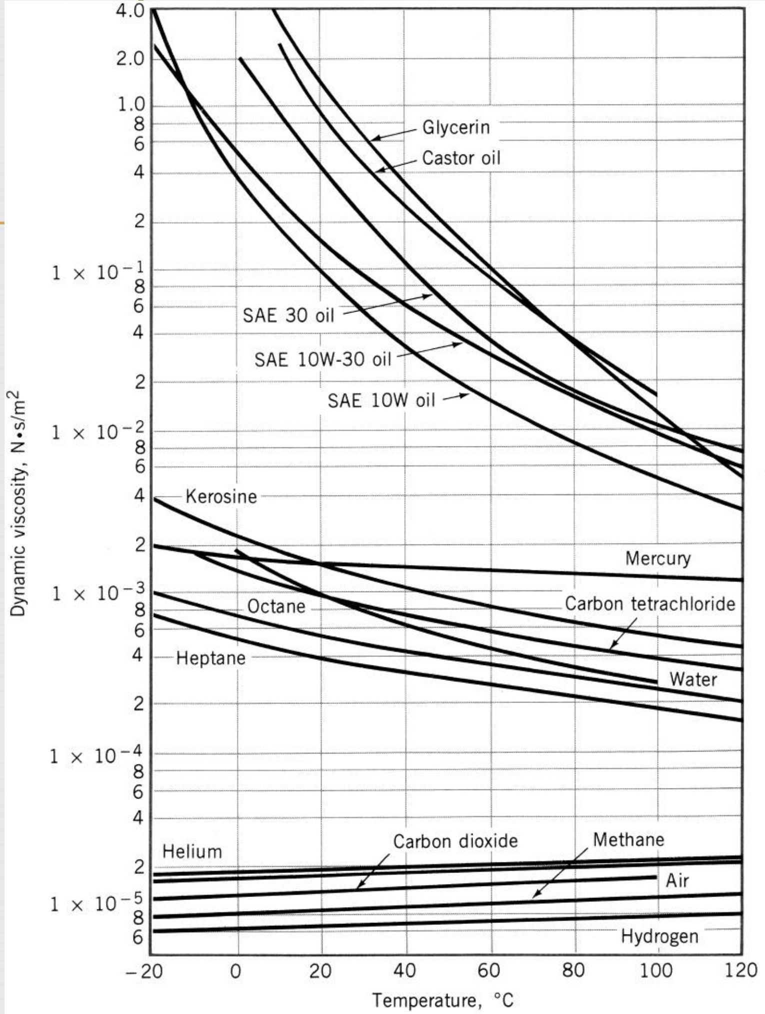
$$\mu = \frac{KT^{3/2}}{T+C}$$

For air: $K=1.448\text{x}10^{-6}$Pa.s/$K^{1/2}$ C=110.4K

- Dynamic viscosity of gases generally increases with temperature

- For liquids, it is the other way round

- Very frequently, we encounter *kinematic viscosity* (i.e. ν), instead of *dynamic viscosity* (i.e. μ)



**Fox R. W. and McDonald A. T. "Introduction to Fluid Mechanics,"*

Fluid viscosity

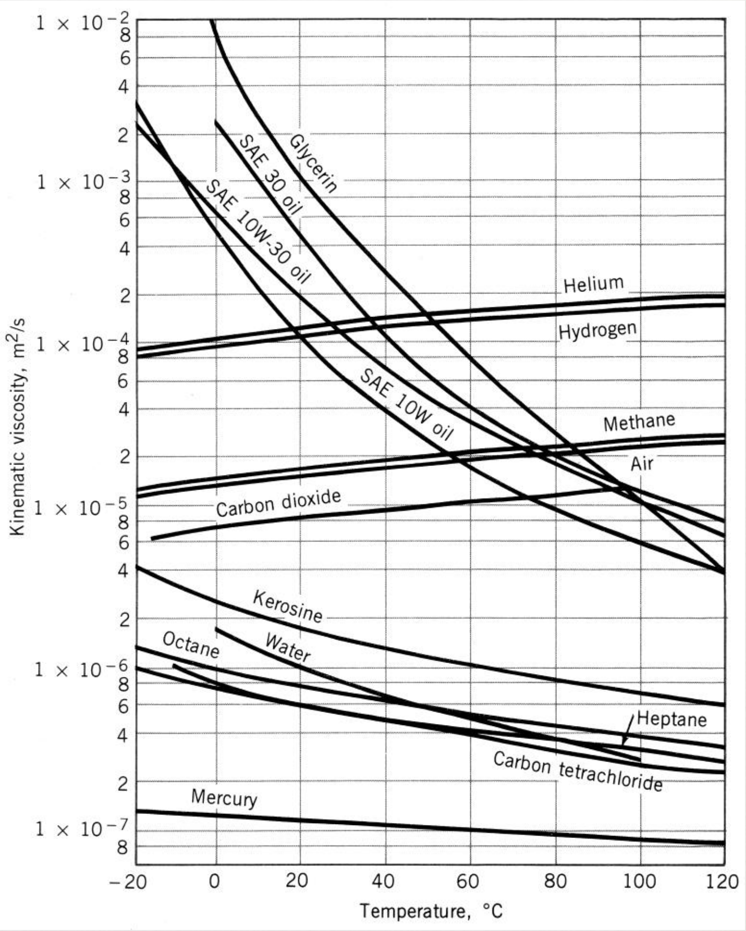
- Kinematic viscosity is defined as

$$\nu = \frac{\mu}{\rho}$$

- Simplistically speaking, it is the ratio of dynamic viscosity to density

- Since density varies with temperature as well, changes in kinematic viscosity differ from those in dynamic viscosity seen earlier

- It should be clear that two fluids having similar kinematic viscosities do no mean that they must have the same dynamics viscosity



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Fluids

- Definition:
  - ✓ A fluid is a substance which is capable of flowing
  - ✓ A fluid is a substance which deform continuously when subjected to external shearing force
- ❖ Characteristics of Fluids:
  - ✓ It has no definite shape of its own, but conforms to the shape of the containing vessel
  - ✓ Even a small amount of shear force exerted on a liquid/fluid will cause it to undergo a deformation which continues as long as the force continues to be applied

Types of fluids

General classification :

- Liquid
- Gas
- Vapour

Based on viscosity

- Ideal
- Real

Ideal fluid	Real fluid
Ideal fluids have zero viscosity.	Viscosity exists.
Incompressible.	Can be compressible.
Infinite bulk modulus	Finite bulk modulus
No surface tension	Surface tension exists
Imaginary and do not exists in nature	Exists in nature

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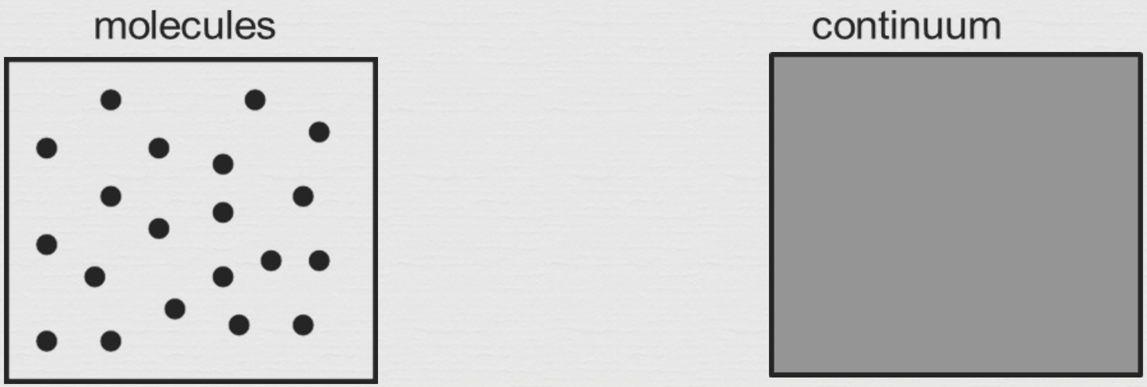

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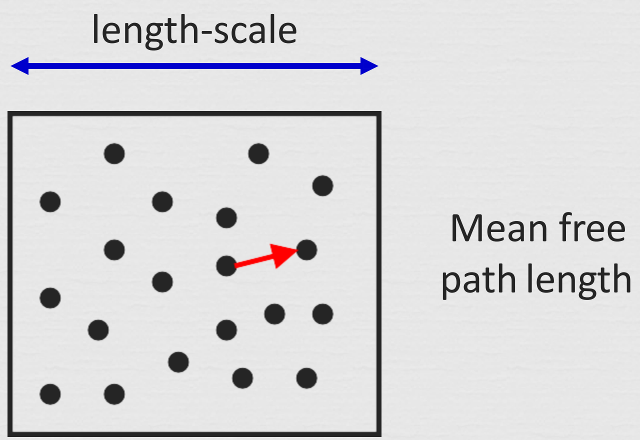

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  - Unit is clearly kg/m^3
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$$\nu = \frac{1}{\rho}$$

  - Pressure, temperature, density are related by the Ideal Gas Equation:

$$pV = mRT$$

$$\Rightarrow p = \rho RT$$

Common fluid properties

- specific weight:
  - The specific weight, also known as the unit weight, is the weight per unit volume of a material.
  - A commonly used value is the specific weight of water on Earth at 4°C, which is 9.807 kN/m^3 or 62.43 lbf/ft^3.
  - The terms specific gravity, and less often specific weight, are also used for relative density

Common fluid properties

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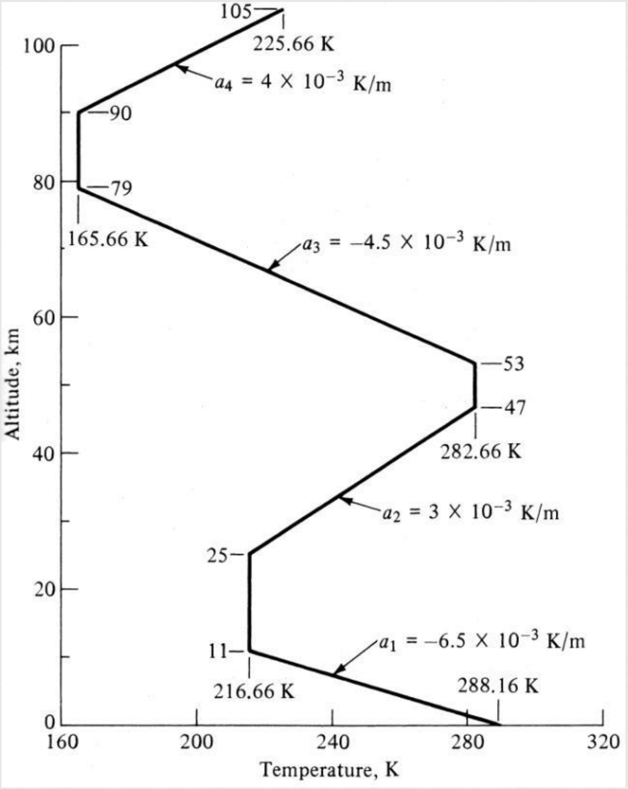

Common fluid properties

Vapour pressure

- Vapour pressure is a measure of the tendency of a material to change into the gaseous or vapour state, and it increases with temperature. The temperature at which the vapour pressure at the surface of a liquid becomes equal to the pressure exerted by the surroundings is called the boiling point of the liquid.

Fluid viscosity

- Viscosity may be defined as the property of a fluid which determines its resistance to shearing stress. It’s a measure of the internal fluid friction which causes resistance to low.
- It is primarily due to cohesive and molecular momentum exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluids.
- The shear stress is proportional to the rate of change of velocity with respect to y

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu . \frac{du}{dy}$$

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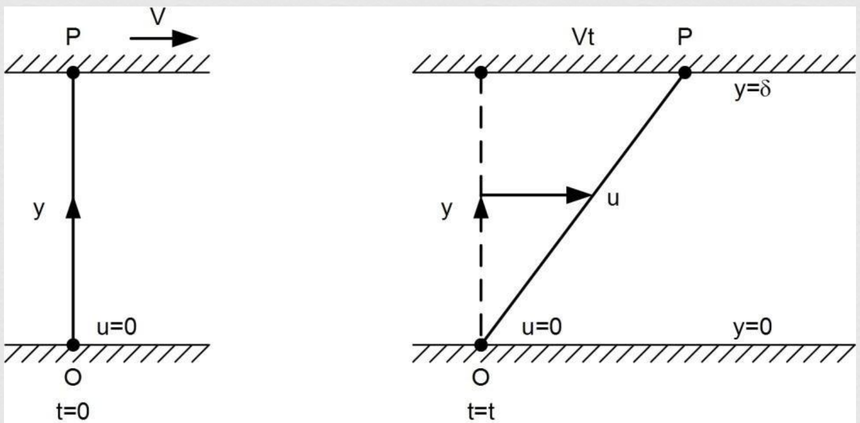

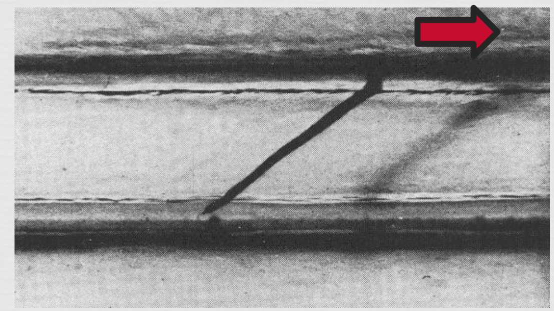

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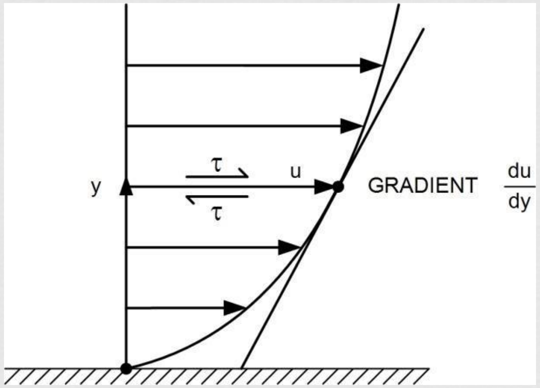

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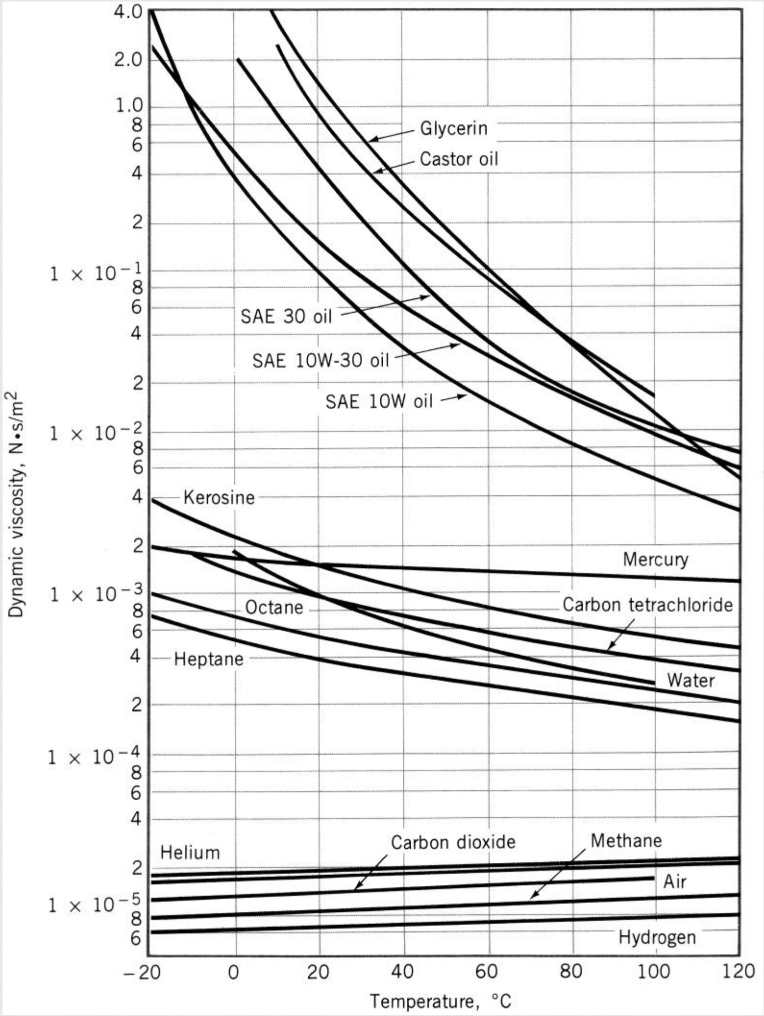

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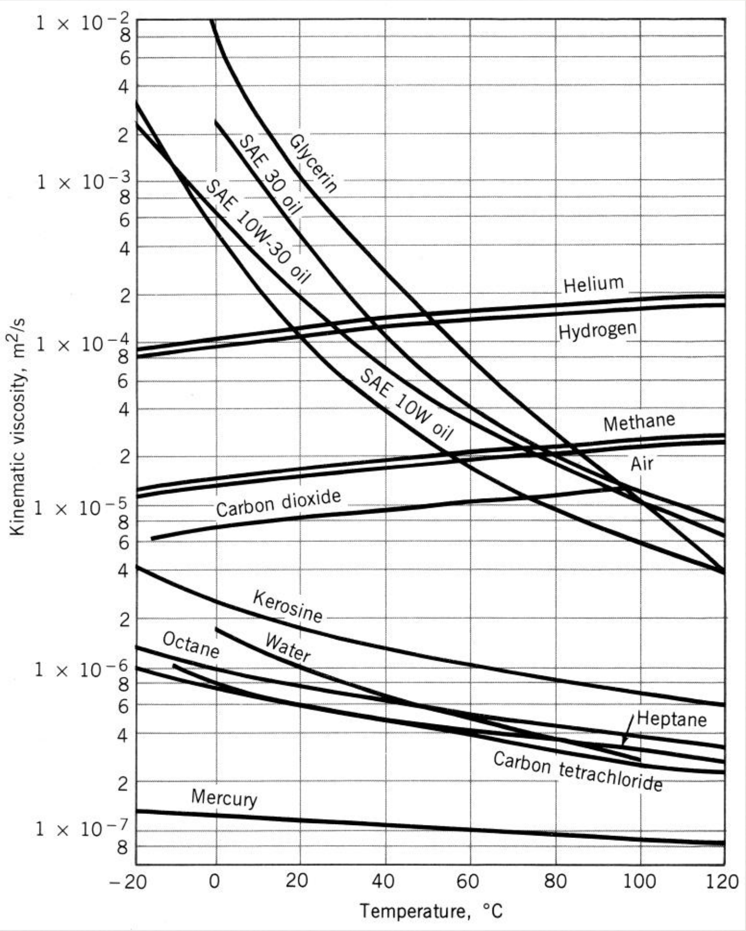

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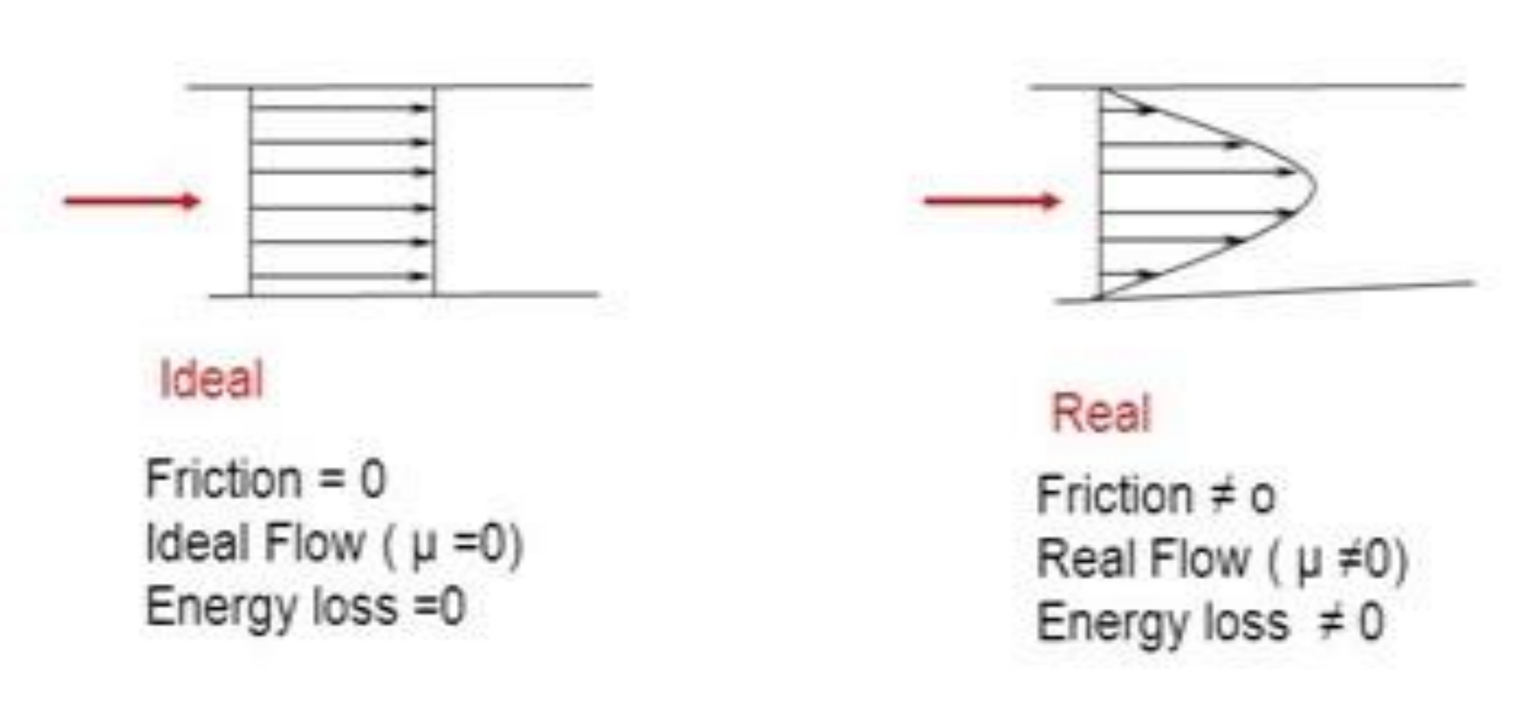
Types of Fluids

- Ideal fluid
- Real fluid
- Newtonian fluid
- Non-Newtonian fluid
- Ideal plastic fluid

Ideal fluid Real fluid

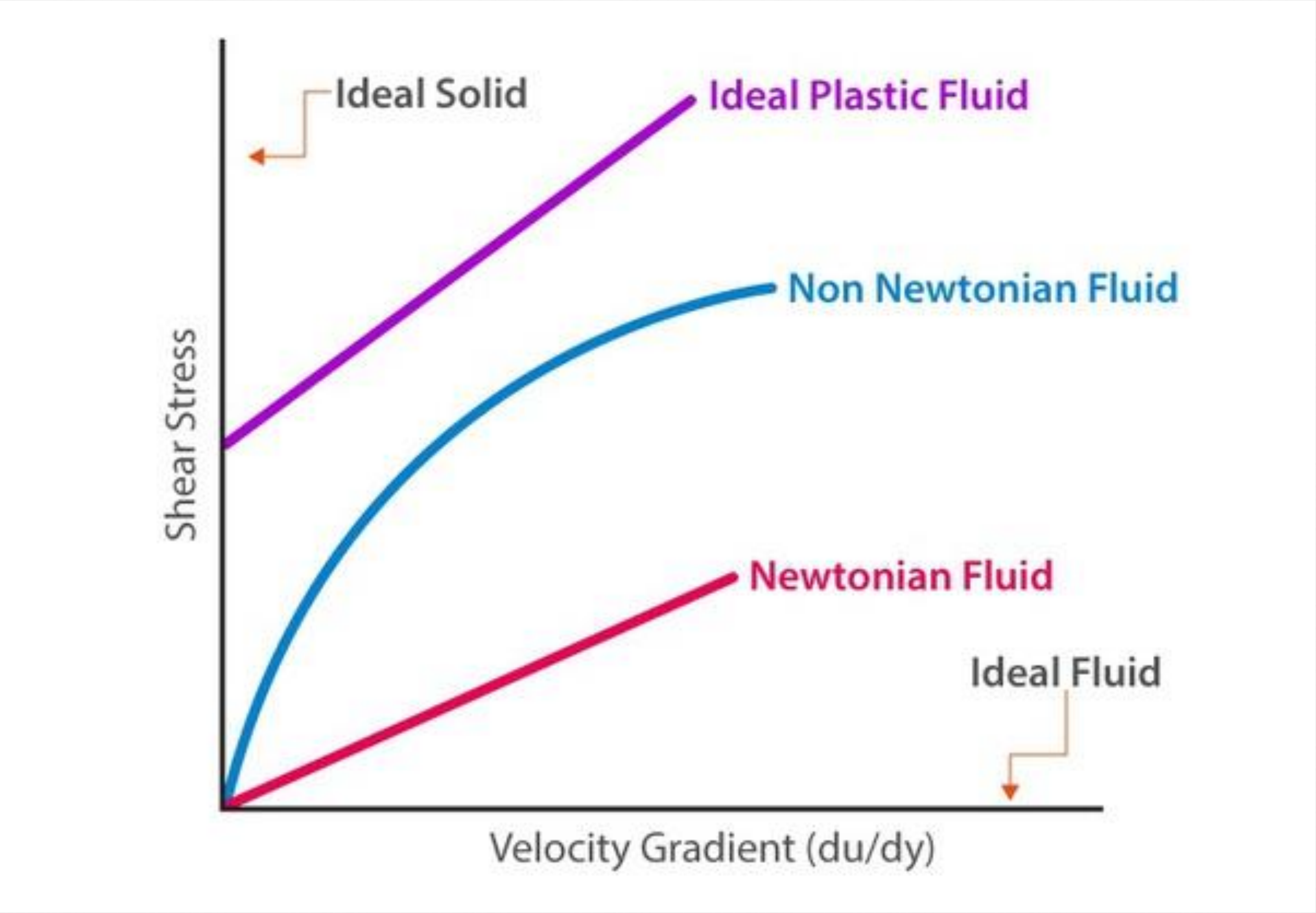
- An ideal fluid (also called Perfect Fluid) is one that is incompressible and has no viscosity. Ideal fluids do not actually exist, but sometimes it is useful to consider what would happen to an ideal fluid in a particular fluid flow problem in order to simplify the problem.
- **Real fluid**: **Fluid** that have viscosity($\mu > 0$) and their motion known as viscous flow. All the **fluids** in actual practice are **real fluids**.

Ideal fluid Real fluid



Newtonian fluid and Non-Newtonian fluid

- A **Newtonian fluid** is **defined** as one with constant viscosity, with zero shear rate at zero shear stress, that is, the shear rate is directly proportional to the shear stress.
- A non-Newtonian fluid is a fluid that does not follow Newton's law of viscosity, i.e., constant viscosity independent of stress. In non-Newtonian fluids, viscosity can change when under force to either more liquid or more solid


Ideal Solid
Ideal Plastic Fluid
Non Newtonian Fluid
Shear Stress
Newtonian Fluid
Ideal Fluid
Velocity Gradient (du/dy)

Ideal plastic fluid

- A real fluid, in which the shear stress is more that the yield value and the shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

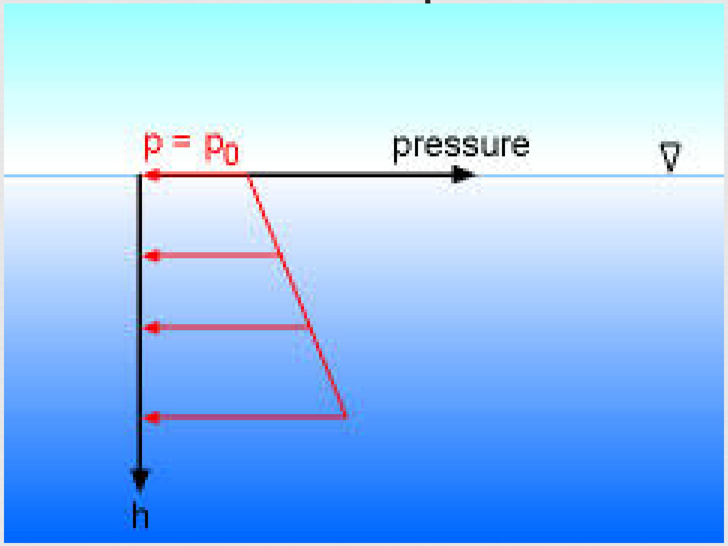
Static fluid

Many fluid problems do not involve motion rather concerned with the pressure distribution in a static fluid. When the fluid velocity is zero, known as hydrostatic condition, the pressure variation is due to weight of the fluid. The important areas of fluid statics include;

- ✓ Pressure distribution in atmospheres and oceans
- ✓ Design of manometer pressure instruments
- ✓ Forces on submerged flat and curved surfaces
- ✓ Buoyancy on a submerged body
- ✓ Behavior of floating bodies

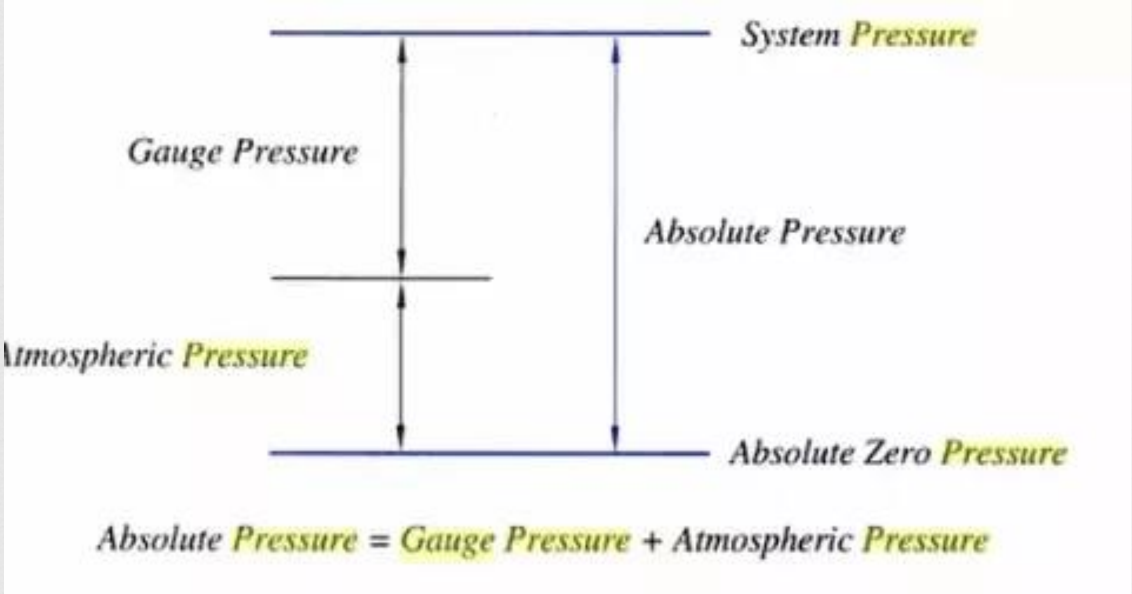
Static fluid

- Fluid pressure is normal force per unit area and its compressive in nature for a static fluid. As there is no shear stress mohar circle is a point



Gauge pressure

- Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

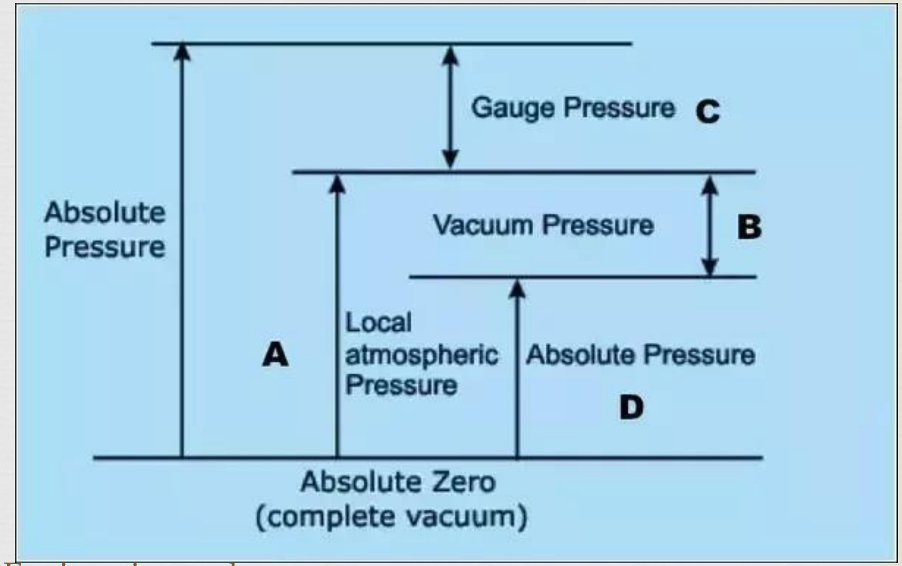


Absolute pressure

- Absolute pressure is the sum of gauge pressure and atmospheric pressure. For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero.
- For finding absolute pressure we will take local atm pressure into account

vacuum pressure

- vacuum pressure is the difference between the atmospheric pressure and the absolute pressure.
- Vacuum pressure is measured relative to ambient atmospheric pressure. It is referred to as pounds per square inch (vacuum) or PSIV



Hydro static law

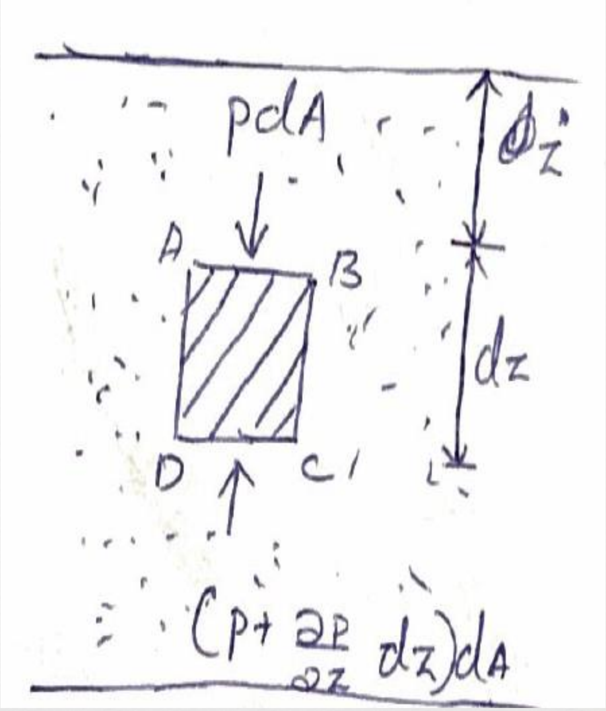
- The Pressure at any point in a static fluid is given by hydrostatic law, which states that, “the rate of increase of pressure in vertical downward direction must be equal to the specific weight of the fluid at that point.

$$\frac{\partial p}{\partial z} = \rho g = w$$

Proof for Hydro static law

Consider a fluid element of cross sectional area dA. Let P is the pressure acting in a face AB in vertical downward direction. Then

$(p + \frac{\partial p}{\partial z} dz) dA$ is the pressure force acting on the side CD in vertical upward direction.



z = distance from surface of fluid.

Now consider the various forces acting on fluid elements are:

* pressure force acting on face AB = $p\,dA$

* pressure acting on CD = $\left(p + \frac{\partial p}{\partial z} dz\right) dA$

* pressure acting on BC and AD are equal, but is opposite directⁿ, hence it get cancelled.

* weight of fluid element.

Now resolve various forces acting on fluid element.

$$-p\,dA + \left(p + \frac{\partial p}{\partial z} dz\right) dA - w = 0 \quad \text{---} (1)$$

$$\rho = \frac{m}{V}$$

$$m = \rho V$$

$$m = \rho\, dA \cdot dz$$

$$W = mg$$

$$W = \rho\, dA\, dz \cdot g \quad \text{——} \ (2)$$

(2) in (1)

$$-pdA + pdA + \frac{\partial p}{\partial z} dz\, dA - \rho\, dA\, dz\, g = 0$$

$$dA\, dz \left[\frac{\partial p}{\partial z} - \rho g\right] = 0$$

$$\frac{\partial p}{\partial z} - \rho g = 0$$

i.e. $$\frac{\partial p}{\partial z} = \rho g$$

$$\frac{\partial p}{\partial z} = \rho g$$

$$\partial p = \rho g\, dz$$

Taking integral

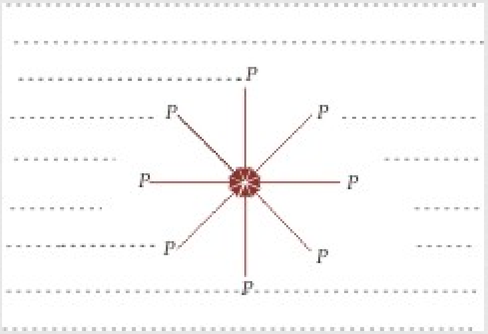
$$\int \partial p = \int \rho g\, dz$$

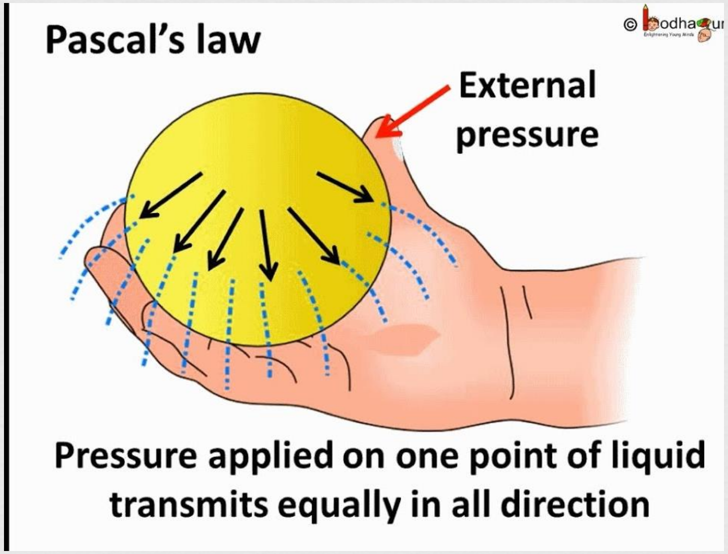
$$p = \rho g z$$

Pascal's law

- Pascal's law says that pressure applied to an enclosed fluid will be transmitted without a change in magnitude to every point of the fluid and to the walls of the container. The pressure at any point in the fluid is equal in all directions.

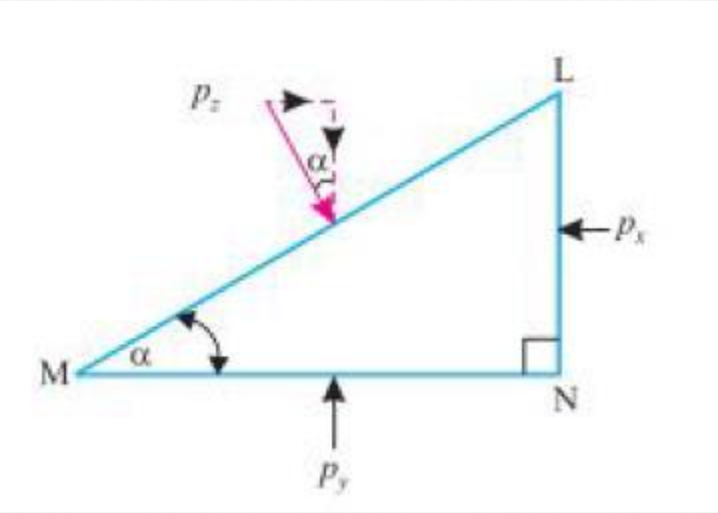
$$P_x = P_y = P_z$$





Pascal's law Proof

$$P_x = p_x \times LN$$
$$P_y = p_y \times MN$$
$$P_z = p_z \times LM$$



$$P_z \sin\alpha = P_x$$

$$p_z . LM . \sin\alpha = p_x . LN \qquad (\because P_z = p_z . LM)$$

But, $LM . \sin\alpha = LN$

$\therefore \quad p_z = p_x$

Resolving the forces *vertically*:

$$P_z . \cos\alpha = P_y - W$$

(where, W = weight of the liquid element)

Since the element is very small, neglecting its weight, we have:

$$P_z \cos\alpha = P_y \quad \text{or} \quad p_z . LM \cos\alpha = p_y . MN$$

But, $LM \cos\alpha = MN$

$\therefore \quad p_z = p_y$

From (*iv*) and (*v*), we get: $p_x = p_y = p_z$

Pressure measuring devices

The pressure of the fluid is measured by the following devices

- Mano meters
- Mechanical gauges

Mano meters

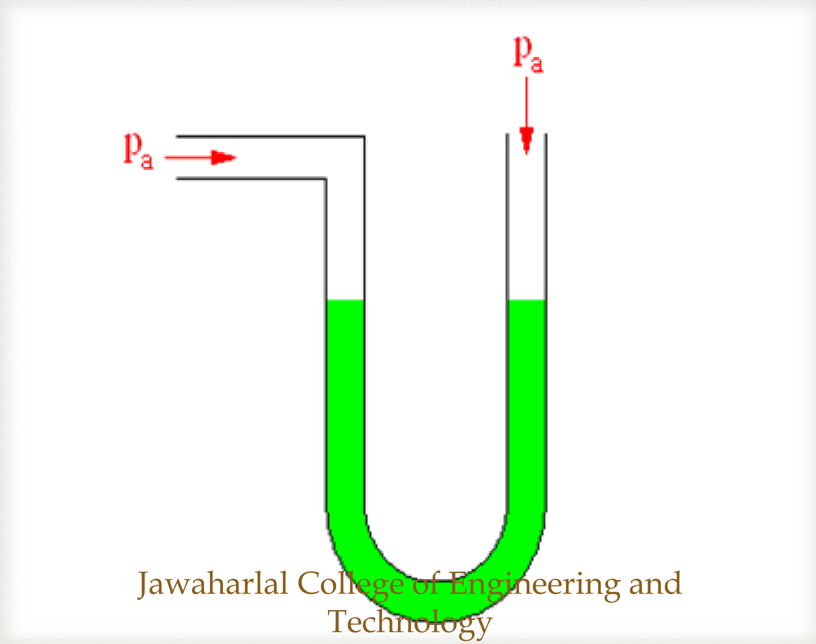
- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same per another column of the fluid. They are classified as
  - Simple manometers
  - Differential Manometers

Mechanical Gauges

- Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are
  - Diaphragm pressure gauge
  - Bourdon tube pressure gauge
  - Dead-weight pressure gauge
  - Bellows pressure gauge

Manometer

- Manometer is a device used for measuring the pressure at a point in a fluid by balancing the column of fluid with the same column or another of the fluid.



Classification of Manometers :

Simple manometer:

- Piezometer
- U-tube manometer
- single column manometer
  - Vertical single column manometer
  - Inclined single column manometer

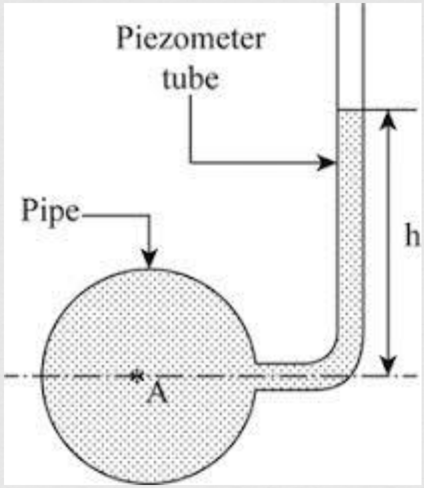
Differential manometer :

- U-tube differential manometer
- Inverted U-tube differential manometer

Piezometer:

A piezometer is the simplest form of the manometer. It measures gauge pressure only.

The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can read on the calibrated scale on glass tube.



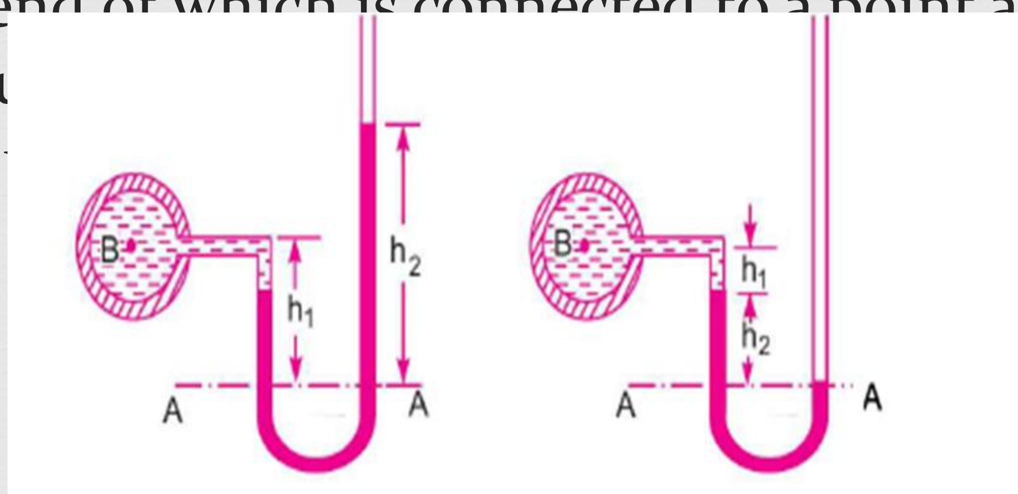
The pressure at point A is given by;

$$p = \rho g h = wh$$

$$\therefore h = \frac{p}{\rho g} \ \textit{piezometric head}$$

U-tube Manometer

- It is used to measure large pressure or vacuum pressure and gas
- It consists of glass tube bent in U-shape
- One end of which is connected to a point at which pressu[illegible] emains open [illegible]



(a) For gauge pressure (b) For vacuum pressure

For a gauge pressure

$$Pressure\ at\ XX\ in\ left\ \ column = Pressure\ at\ XX\ in\ \text{right}\ column$$

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$p = \rho_2 g h_2 - \rho_1 g h_1$$

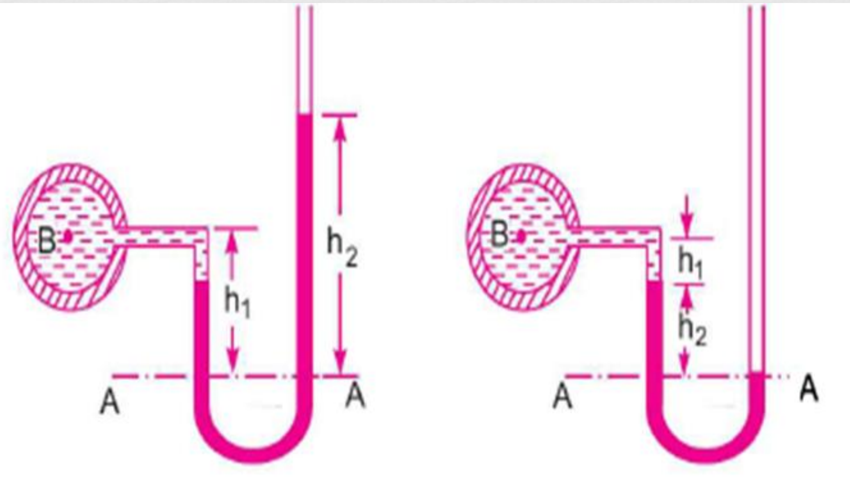
For vacuum pressure

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$



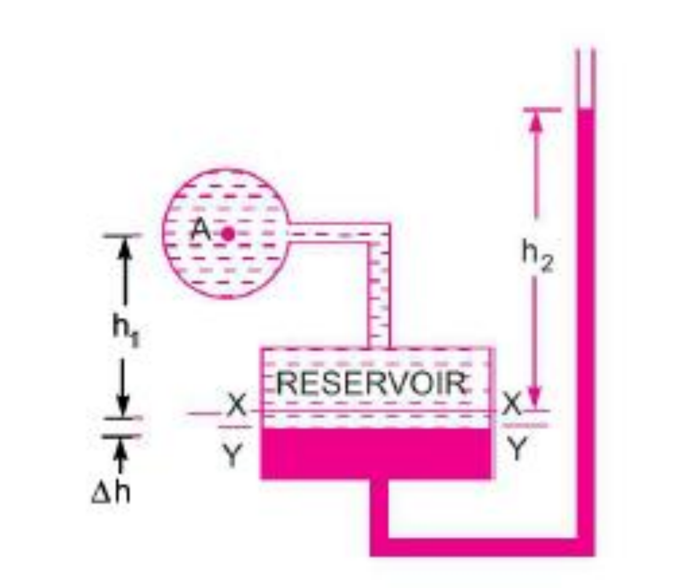
(a) For gauge pressure (b) For vacuum pressure

Single column Manometer

- One of the limbs in double column manometer is converted into reservoir having large cross sectional area (about 100 times) with respect to the otherlimb.
- Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence pressure is given by the height of the liquid in the other limb.
  - Vertical single column manometer
  - Inclined single column manometer

(A) Vertical single column manometer

$Volume\ of\ heavy\ liquid\ fall\ in\ reservoir$
$= Volume\ of\ heavy\ liquid\ rise\ in\ right\ column$



$$\therefore A \times \Delta h = a \times h_2 \rightarrow \Delta h = \frac{a \times h_2}{A}$$

Pressure in left col. = pressure in right col.

Now consider the datum line Y-Y as shown in Fig. 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

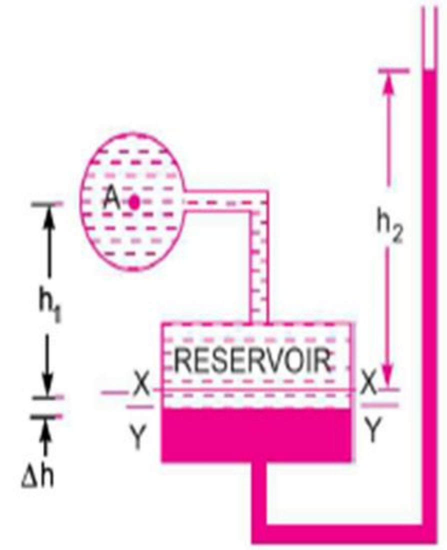
or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (i), $\Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho$$



Final Equation

$$P_A = h_2\rho_2 g - h_1\rho_1 g$$

Inclined single column manometer

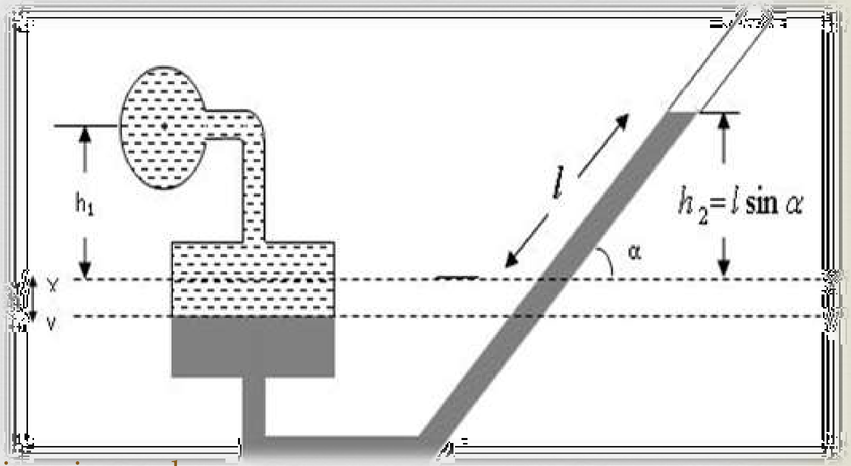
It is modified of vertical column manometer. This manometer is useful for the measurement of small pressure.

Here, $height\ h_2 = l\ sin\theta\ and\ putting\ in\ above\ eq.\ (i);$

$$\therefore p = \frac{al\ sin\theta}{A}[\rho_2 g - \rho_1 g] + \rho_2 glsin\theta - \rho_1 gh_1$$

- $since,\ a \ll A,\ neglecting\ first$

- $\therefore p = \rho_2 glsin\theta - \rho_1 gh_1$



U-tube differential manometer

It is used to measure pressure difference at two points in a pipe or between two pipes at different levels.

Case 1 - U-tube upright differential manometer connected at two points in a pipe at different level

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

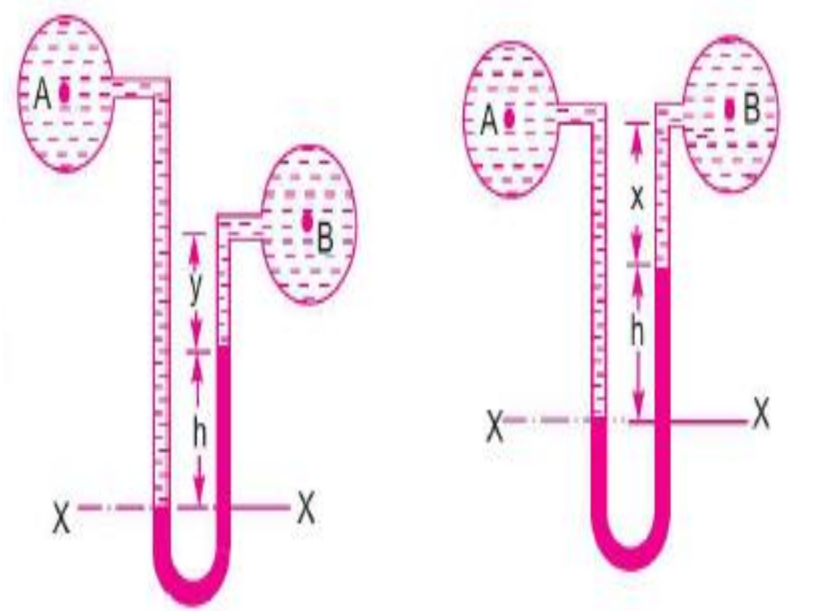
Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

$\therefore$ Difference of pressure at A and B $= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$



(a) Two pipes at different levels (b) A and B are at the same level

Case 2 - U-tube upright differential manometer connected between two pipes at same levels and carrying different fluids

In Fig. 2.18 (*b*), the two points *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h + x)$$

$$= g \times h(\rho_g - \rho_1). \quad \text{...(2.13)}$$

Inverted U-tube differential manometer

It is used for low pressure difference.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

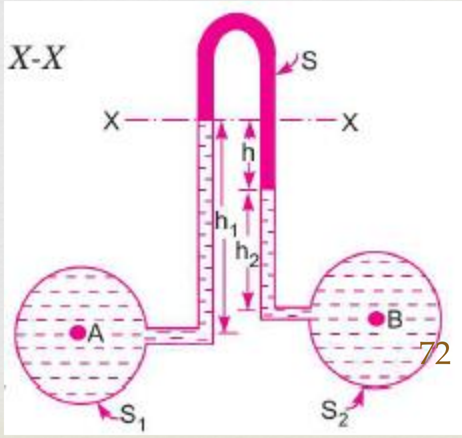
$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

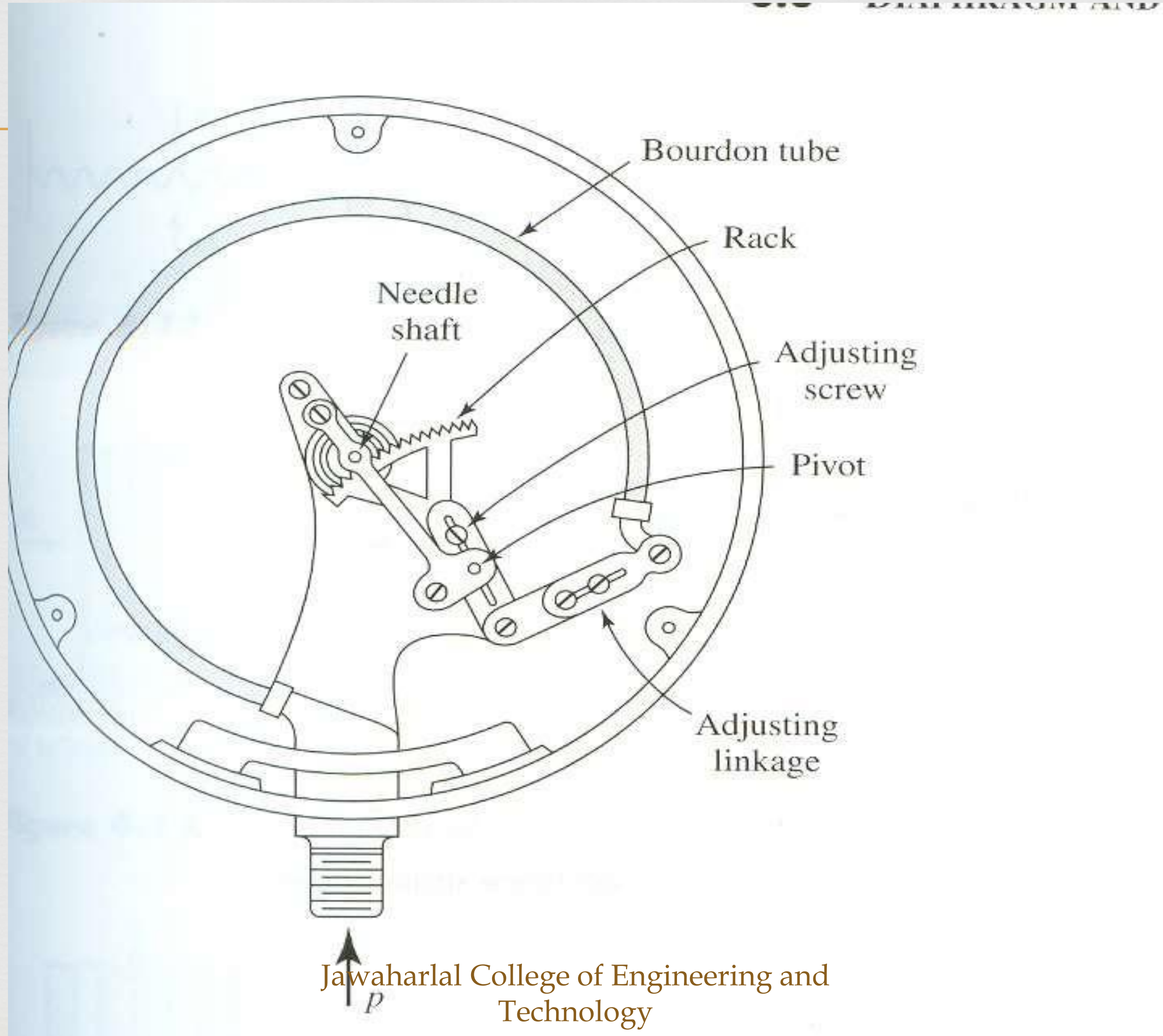
$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$



Bourdon Tube



bourdon tube.MKV

- The linkage is constructed so that the mechanism m be adjusted for optimum linearity and minimum hysteresis as well as compensate for wear which may develop over a period of time.
- An electrical-resistance strain gauge may also be installed on the bourdon-tube to sense thr elastic deformation.

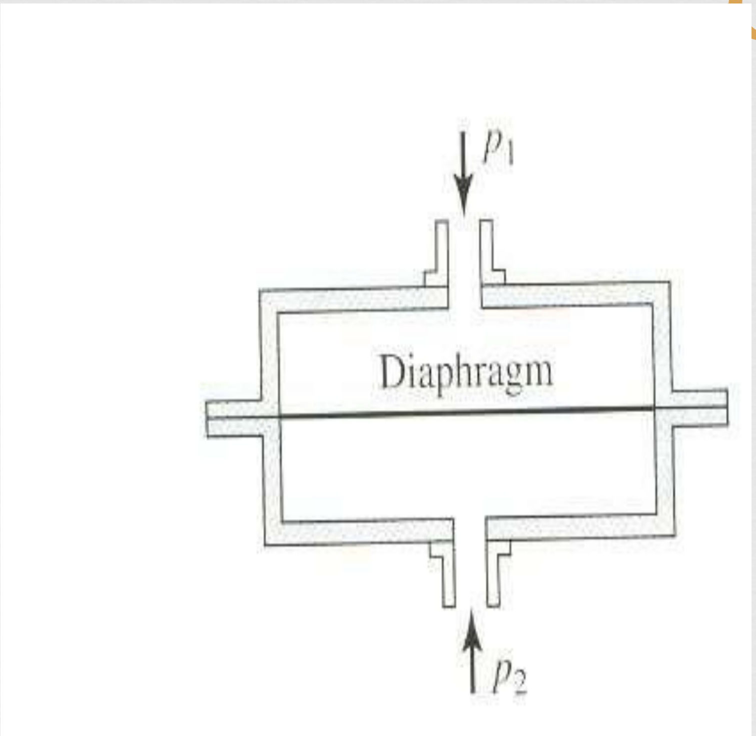
4) Diaphragm and Bellows Gauges

- Represent similar types of elastic deformation devices useful for pressure measurementapplications.
- Architecture and operation:

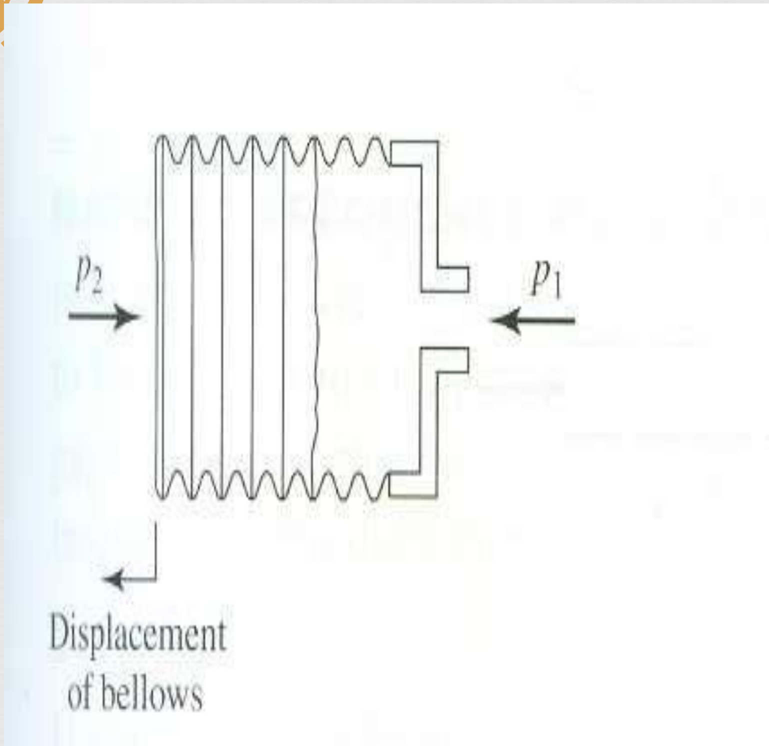
Diaphragm gauge:

- Consider first the flat diaphragm subjected to the differential pressure p1-p2 as shown in figure .
- The diaphragm will be deflected in accordance with this pressure differential and the deflection sensed an appropriate displacementtransducer.
- Various types ofdiaphragm gauge are shown figure 4.10

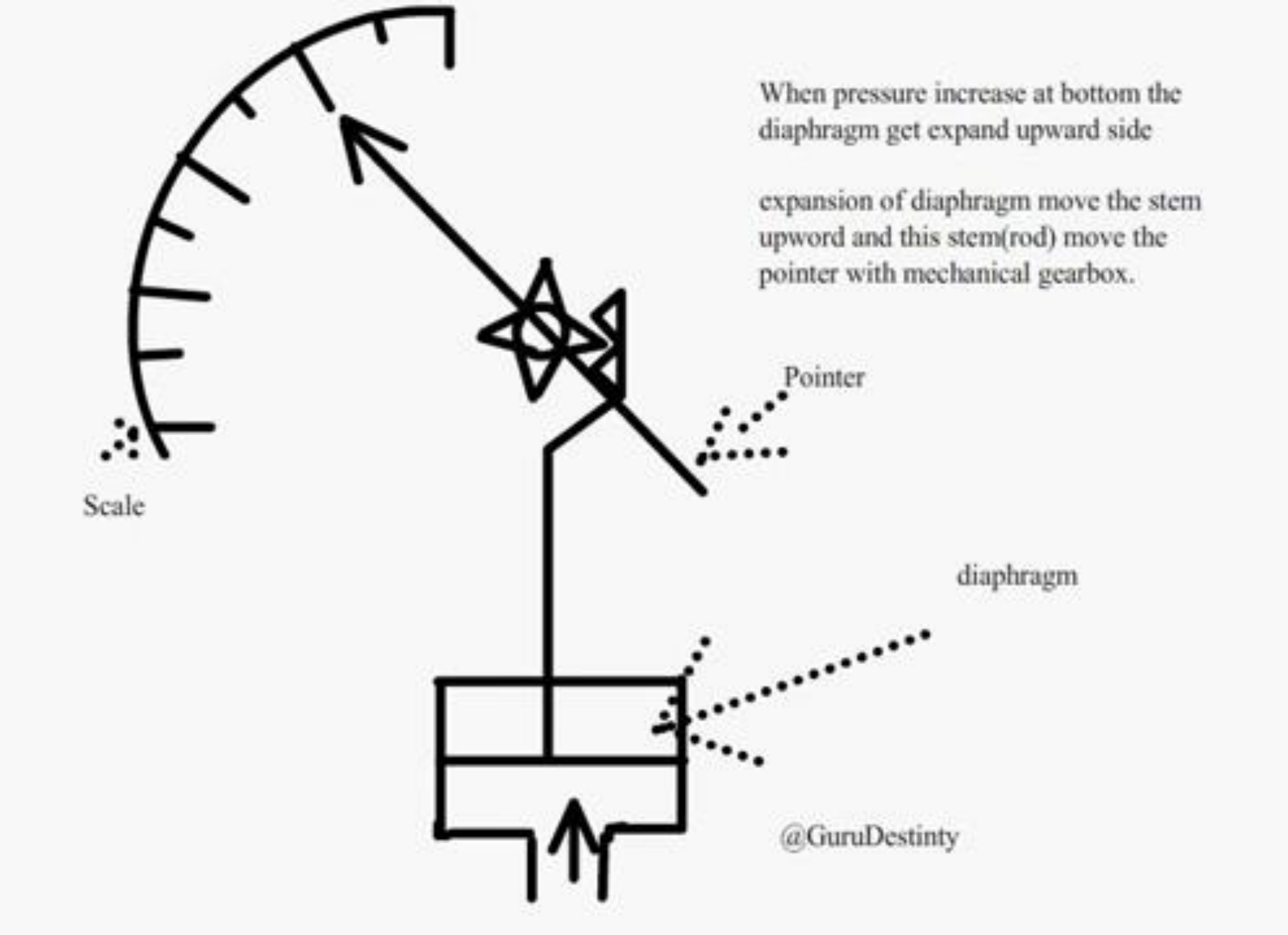
(a) Diaphragm and (b) Bellows



(a)



(b)


When pressure increase at bottom the diaphragm get expand upward side

expansion of diaphragm move the stem upword and this stem(rod) move the pointer with mechanical gearbox.
Pointer
Scale
diaphragm
@GuruDestinty

Bellows Gauge:

- The bellows gauge is shown in figure (b).
- A differential gauge pressure force causes displacement of the bellows, which may be converted to an electrical signal or undergo a mechanical amplification to permit display of the output on an indicator dial.
- Figure shows various types of bellowsgauges.

- The bellows gauge is generally unsuitable for transient measurements because of the larger relative motion and mass involved.
- The diaphragm gauge which may be quite stiff, involves rather small displacements and is suit for high frequency pressure measurement.

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m^3.

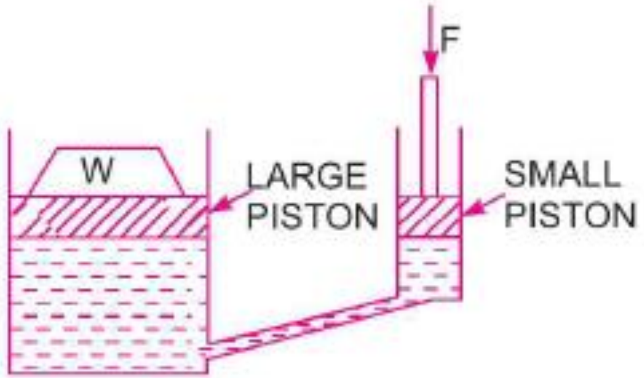


Fig. 2.5

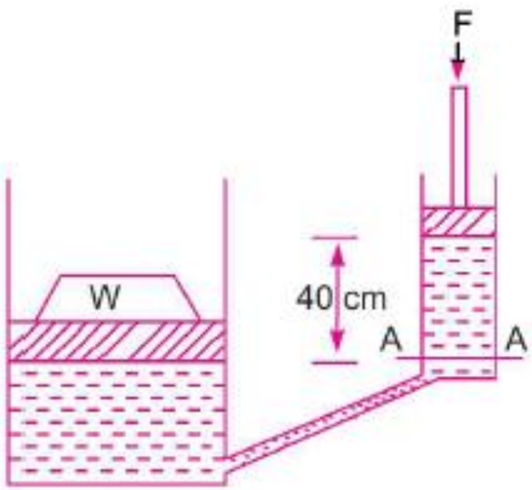


Fig. 2.6

Dia. of large piston, $D = 10$ cm

∴ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(a) When the pistons are at the same level

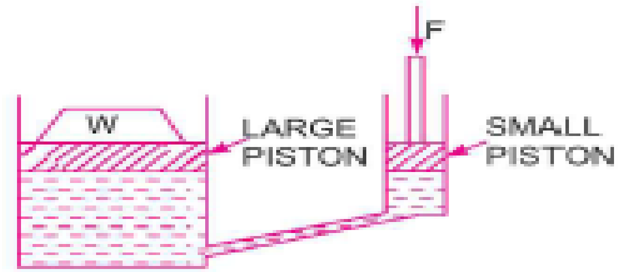


Fig. 2.5

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

This is transmitted equally on the large piston.

∴ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

∴ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

(b) When the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

∴ Pressure intensity at section A-A

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

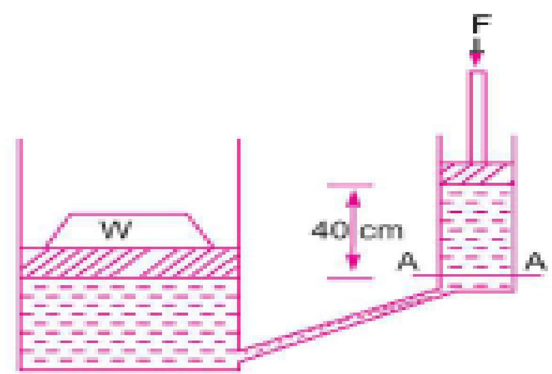


Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

∴ Pressure intensity at section A-A

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

∴ Pressure intensity transmitted to the large piston $= 11.71 \text{ N/cm}^2$

∴ Force on the large piston $=$ Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

∴ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

∴ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from A-A, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

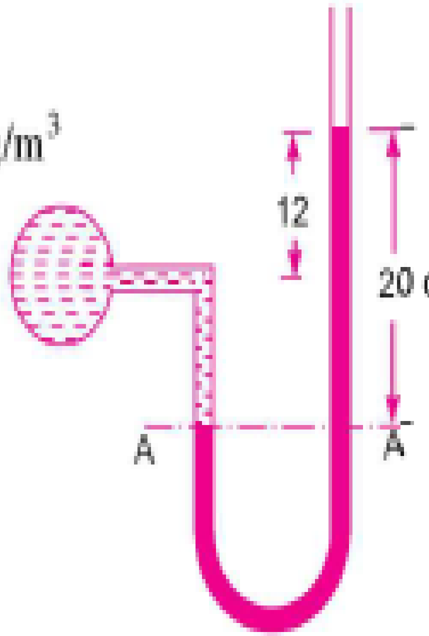


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Problem 2.13 *A pressure gauge consists of two cylindrical bulbs B and C each of 10 sq. cm cross-sectional area, which are connected by a U-tube with vertical limbs each of 0.25 sq. cm cross-sectional area. A red liquid of specific gravity 0.9 is filled into C and clear water is filled into B, the surface of separation being in the limb attached to C. Find the displacement of the surface of separation when the pressure on the surface in C is greater than that in B by an amount equal to 1 cm head of water.*

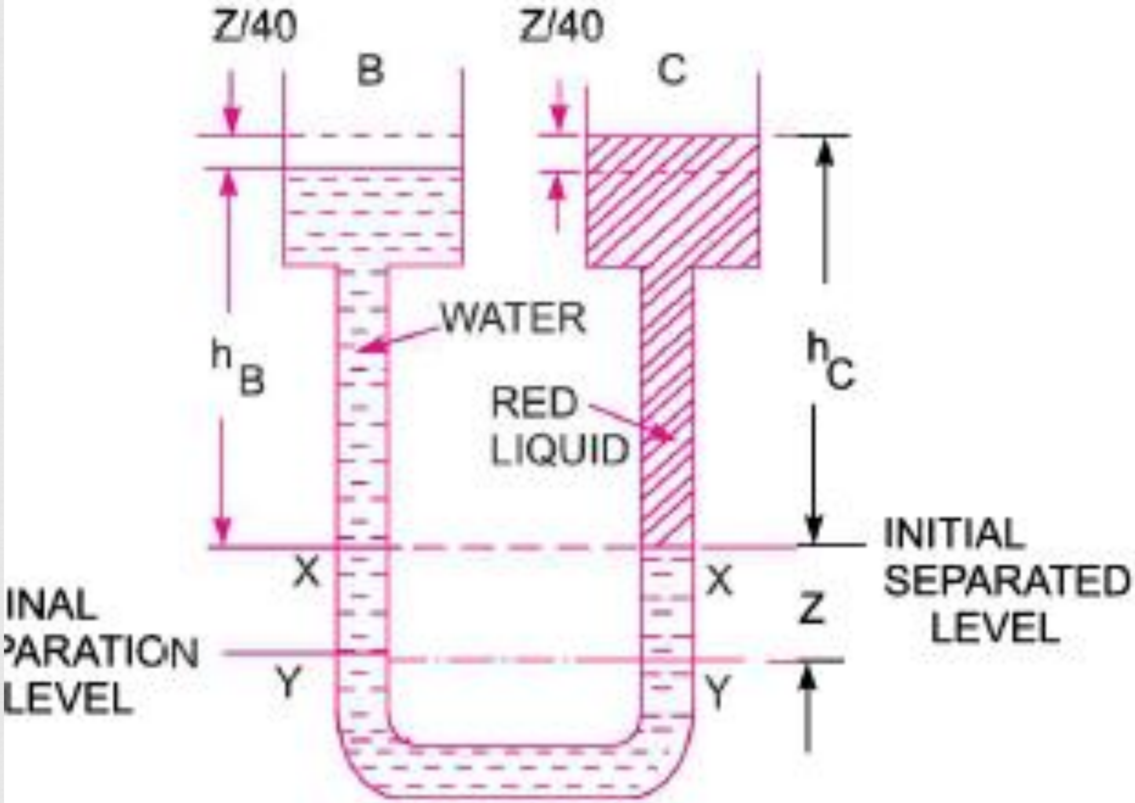

Z/40
B
Z/40
C
WATER
h_B
RED
LIQUID
h_C
INITIAL
SEPARATED
LEVEL
X
X
Z
INAL
ARATION
LEVEL
Y
Y

Fig. 2.14

Solution. Given :

Area of each bulb B and C, $A = 10 \text{ cm}^2$

Area of each vertical limb, $a = 0.25 \text{ cm}^2$

Sp. gr. of red liquid $= 0.9$ $\therefore$ Its density $= 900 \text{ kg/m}^3$

Let $X\text{-}X$ = Initial separation level

h_C = Height of red liquid above $X\text{-}X$

h_B = Height of water above $X\text{-}X$

Pressure above $X\text{-}X$ in the left limb $= 1000 \times 9.81 \times h_B$

Pressure above $X\text{-}X$ in the right limb $= 900 \times 9.81 \times h_C$

Equating the two pressure, we get

$$1000 \times 9.81 \times h_B = 900 \times 9.81 \times h_C$$

$$\therefore \quad h_B = 0.9\, h_C \quad \ldots(i)$$

When the pressure head over the surface in C is increased by 1 cm of water, let the separation level falls by an amount equal to Z. Then $Y\text{-}Y$ becomes the final separation level.

Now fall in surface level of C multiplied by cross-sectional area of bulb C must be equal to the fall in separation level multiplied by cross-sectional area of limb.

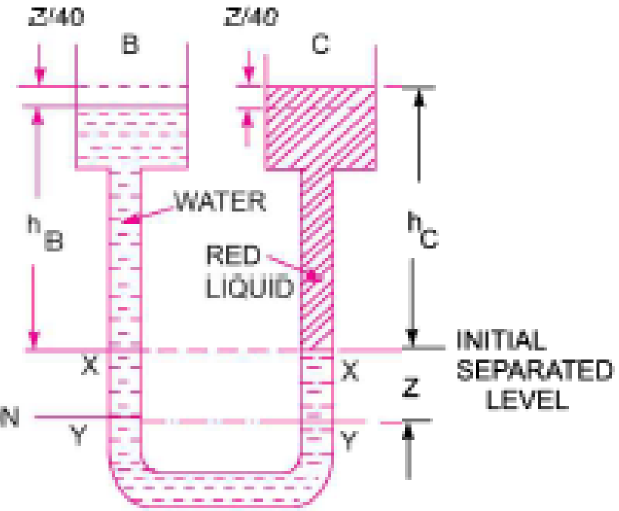


Fig. 2.14

$\therefore$ Fall in surface level of C

$$= \frac{\text{Fall in separation level} \times a}{A}$$

$$= \frac{Z \times a}{A} = \frac{Z \times 0.25}{10} = \frac{Z}{40}.$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water = $\rho g h = 1000 \times 9.81 \times 0.01 = 98.1$ N/m^2

Consider final separation level Y-Y

Pressure above Y-Y in the left limb = $1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right)$

Pressure above Y-Y in the right limb = $900 \times 9.81 \left(Z + h_C - \frac{Z}{40}\right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right) = \left(Z + h_C - \frac{Z}{40}\right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40}\right) = 900 \left(Z + h_C - \frac{Z}{40}\right) + 10$$

Dividing by 1000, we get $Z + h_B + \frac{Z}{40} = 0.9 \left(Z + h_C - \frac{Z}{40}\right) + 0.01$

But from equation (i), $\quad h_B = 0.9\, h_C$

$$\therefore \quad Z + 0.9\, h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\, h_C + 0.01$$

or

$$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or

$$Z \left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z \left(\frac{41 - 35.1}{40}\right) = .01$$

$$\therefore \quad Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

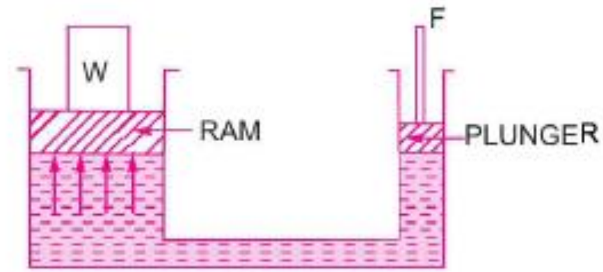


Fig. 2.3

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

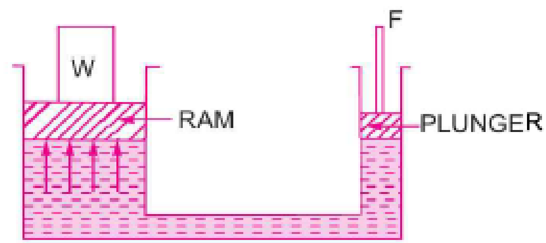


Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} =$ **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20 \text{ cm} = 0.2 \text{ m}$

∴ Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3 \text{ cm} = 0.03 \text{ m}$

∴ Area of plunger, $a = \frac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000 \text{ N}.$

See Fig. 2.3.

Pressure intensity developed due to plunger $= \frac{\text{Force}}{\text{Area}} = \frac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \frac{F}{a}$

∴ Force acting on ram = Pressure intensity × Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \mathbf{675.2 \text{ N. Ans.}}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, ρ_2 = Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m^3

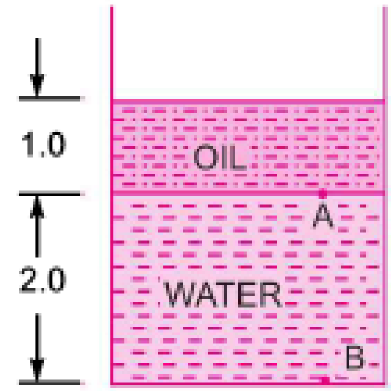


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0 = 900 \times 9.81 \times 1.0 = 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.\ Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \mathbf{2.8449\ N/cm^2.\ Ans.}$$

Problem 2.14 *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

∴ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let p_A = Pressure in pipe

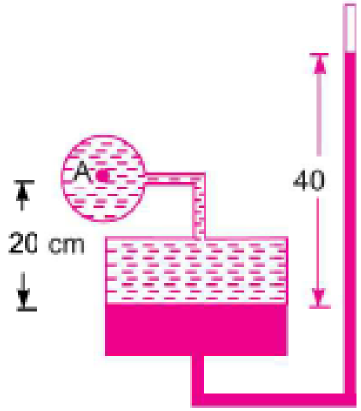


Fig. 2.17

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm². Ans.**

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.*

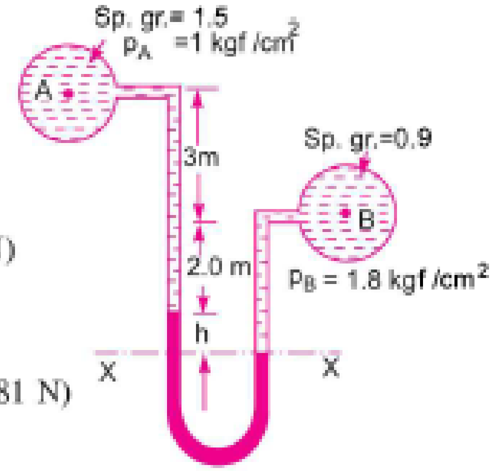


Fig. 2.19

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5 \quad \therefore \quad \rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9 \quad \therefore \quad \rho_2 = 900$

Pressure at A, $p_A = 1 \text{ kgf/cm}^2 = 1 \times 10^4 \text{ kgf/m}^2$

$= 10^4 \times 9.81 \text{ N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8 \text{ kgf/cm}^2$

$= 1.8 \times 10^4 \text{ kgf/m}^2$

$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $\quad 13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$

or $\quad (13.6 - 0.9)h = 19.8 - 17.5$ or $12.7h = 2.3$

$\therefore \quad h = \dfrac{2.3}{12.7} = 0.181 \text{ m} =$ **18.1 cm. Ans.**

Problem 2.19 *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

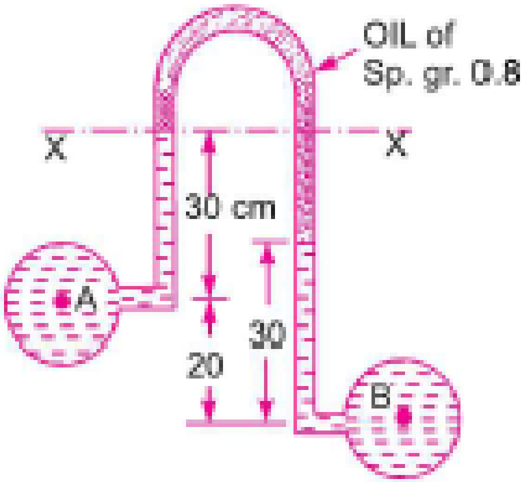


Fig. 2.23

Solution. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$= (30 + 20) - 30 = 20$ cm

Taking datum line at X-X

Pressure in the left limb below X-X

$= p_A - 1000 \times 9.81 \times 0$

$= p_A - 2943$

Pressure in the right limb below X-X

$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= p_B - 2943 - 1569.6 = p_B - 4512.6$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore \quad p_B - p_A = 4512.6 - 2943 = $ **1569.6 N/m². Ans.**

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.*

Pressure at a point in compressible fluid

- For a compressible fluids density changes with change in

$$\frac{p}{\rho} = RT \qquad \text{...(2.15)}$$

or

$$\rho = \frac{p}{RT}$$

Now equation (2.4) is

$$\frac{dp}{dZ} = w = \rho g = \frac{p}{RT} \times g$$

$\therefore$

$$\frac{dp}{p} = \frac{g}{RT} dZ \qquad \text{...(2.16)}$$

In equation (2.4), Z is measured vertically downward. But if Z is measured vertically up, then $\frac{dp}{dZ} = -\rho g$ and hence equation (2.16) becomes

$$\frac{dp}{p} = \frac{-g}{RT} dZ \qquad \text{...(2.17)}$$

Isothermal Progress

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT}\, dz = -\frac{g}{RT}\int_{Z_0}^{Z} dz$$

or $$\log \frac{p}{p_0} = \frac{-g}{RT}[Z - Z_0]$$

where p_0 is the pressure where height is Z_0. If the datum line is taken at Z_0, then $Z_0 = 0$ and p_0 becomes the pressure at datum line.

$\therefore$ $$\log \frac{p}{p_0} = \frac{-g}{RT} Z$$

$$\frac{p}{p_0} = e^{-gZ/RT}$$

or pressure at a height Z is given by $p = p_0 e^{-gZ/RT}$ ...(2.18)

Adiabatic Process

- In adiabatic pro $\frac{p}{\rho^k} = \text{Constant} = C$ tween pressure and density is

where k is ratio of specific constant.

$$\therefore \quad \rho^k = \frac{p}{C}$$

or
$$\rho = \left(\frac{p}{C}\right)^{1/k} \qquad \ldots(ii)$$

Then equation (2.4) for Z measured vertically up becomes,

$$\frac{dp}{dZ} = -\rho g = -\left(\frac{p}{C}\right)^{1/k} g$$

or
$$\frac{dp}{\left(\frac{p}{C}\right)^{1/k}} = -gdZ \text{ or } C^{1/k}\frac{dp}{\frac{1}{p^k}} = -gdZ$$

Integrating, we get $\int_{p_0}^{p} C^{1/k}\, p^{-1/k}\, dp = \int_{Z_0}^{Z} -gdZ$

or
$$C^{1/k}\left[\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

or
$$\left[\frac{C^{1/k}\,p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z} \qquad [C \text{ is a constant, can be taken inside}]$$

But from equation (i), $$C^{1/k} = \left(\frac{p}{\rho^k}\right)^{1/k} = \frac{p^{1/k}}{\rho}$$

Substituting this value of $C^{1/k}$ above, we get

$$\left[\frac{p^{1/k}}{\rho}\,\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -\,g[Z - Z_0]$$

or $$\left[\frac{p^{\frac{1-1+k}{k}}}{\rho^{\frac{k-1}{k}}}\right]_{p_0}^{p} = -\,g[Z - Z_0] \text{ or } \left[\frac{k}{k-1}\,\frac{p}{\rho}\right]_{p_0}^{p} = -\,g[Z - Z_0]$$

or $$\frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -\,g[Z - Z_0]$$

If datum line is taken at Z_0, where pressure, temperature and density are p_0, T_0 and ρ_0, then $Z_0 = 0$.

$\therefore$ $$\frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -\,gZ \quad \text{or} \quad \frac{p}{\rho} - \frac{p_0}{\rho_0} = -\,gZ\,\frac{(k-1)}{k}$$

or $$\frac{p}{\rho} = \frac{p_0}{\rho_0} - gZ\,\frac{(k-1)}{k} = \frac{p_0}{\rho_0}\left[1 - \frac{k-1}{k}\,gZ\,\frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{\rho} \times \frac{\rho_0}{p_0} = \left[1 + \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

But from equation (*i*), $$\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k} \quad \text{or} \quad \left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p} \text{ or } \frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/k}$$

Substituting the value of $\frac{\rho_0}{\rho}$ in equation (*iii*), we get

$$\frac{p}{p_0} \times \left(\frac{p_0}{p}\right)^{1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{p_0} \times \left(\frac{p}{p_0}\right)^{-1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\left(\frac{p}{p_0}\right)^{1-\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

$\therefore$ $$\frac{p}{p_0} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{\rho_0}\right]^{\frac{k}{k-1}}$$

$\therefore$ Pressure at a height Z from ground level is given by

$$p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}}$$

In equation (2.19), p_0 = pressure at ground level, where $Z_0 = 0$

ρ_0 = density of air at ground level

Equation of state is $$\frac{p_0}{\rho_0} = RT_0 \text{ or } \frac{\rho_0}{p_0} = \frac{1}{RT_0}$$

Substituting the values of $\frac{\rho_0}{p_0}$ in equation (2.19), we get

$$p = p_0 \left[1 - \frac{k-1}{k} \frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

Temperature at any point in compressible fluid

- For adiabatic process temperature at any height in air is calculated as

$$\frac{p_0}{\rho_0} = RT_0 \text{ and } \frac{p}{\rho} = RT$$

Dividing these equations, we get

$$\left(\frac{p_0}{\rho_0}\right) \div \frac{p}{\rho} = \frac{RT_0}{RT} = \frac{T_0}{T} \quad \text{or} \quad \frac{p_0}{\rho_0} \times \frac{\rho}{p} = \frac{T_0}{T}$$

or

$$\frac{T}{T_0} = \frac{\rho_0}{p_0} \times \frac{p}{\rho} = \frac{p}{p_0} \times \frac{\rho_0}{\rho}$$

But $\frac{p}{p_0}$ from equation (2.20) is given by

$$\frac{p}{p_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

Also for adiabatic process $\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k}$ or $\left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p}$

or

$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{-\frac{1}{k}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\left(\frac{k}{k-1}\right)\times\left(-\frac{1}{k}\right)} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

Substituting the values of $\frac{p}{p_0}$ and $\frac{\rho_0}{\rho}$ in equation (*i*), we get

$$\frac{T}{T_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \times \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1} - \frac{1}{k-1}} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

$$\therefore \quad T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right] \qquad \text{...(2.21)}$$

2.8.4 Temperature Lapse-Rate (L). It is defined as the rate at which the temperature changes with elevation. To obtain an expression for the temperature lapse-rate, the temperature given by equation (2.21) is differentiated with respect to Z as

$$\frac{dT}{dZ} = \frac{d}{dZ}\left[T_0\left(1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right)\right]$$

where T_0, K, g and R are constant

$$\therefore \qquad \frac{dT}{dZ} = -\frac{k-1}{k} \times \frac{g}{RT_0} \times T_0 = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

The temperature lapse-rate is denoted by L and hence

$$L = \frac{dT}{dZ} = \frac{-g}{R}\left(\frac{k-1}{k}\right) \qquad ...(2.22)$$

In equation (2.22), if (*i*) $k = 1$ which means isothermal process, $\frac{dT}{dZ} = 0$, which means temperature is constant with height.

(*ii*) If $k > 1$, the lapse-rate is negative which means temperature decreases with the increase in height.

In atmosphere, the value of k varies with height and hence the value of temperature lapse-rate also varies. From the sea-level upto an elevation of about 11000 m (or 11 km), the temperature of air decreases uniformly at the rate of 0.0065°C/m. from 11000 m to 32000 m, the temperature remains constant at – 56.5°C and hence in this range lapse-rate is zero. Temperature rises again after 32000 m in air.

Hydrostatic force on surface

- There will be no relative motion between adjacent or neighboring fluid layers the velocity gradient, which is equal to the change of velocity between adjacent fluid layer divi $\frac{du}{dy} = 0.$ he d $\mu \frac{\partial u}{\partial y}$ e between the layers, will be zero.

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

Center of pressure

- The center of pressure is the point where the total sum of a pressure field acts on a body, causing a force to act through that point. The total force vector acting at the cente egrated vectorially press

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

Vertical plan surface submerged in liquid

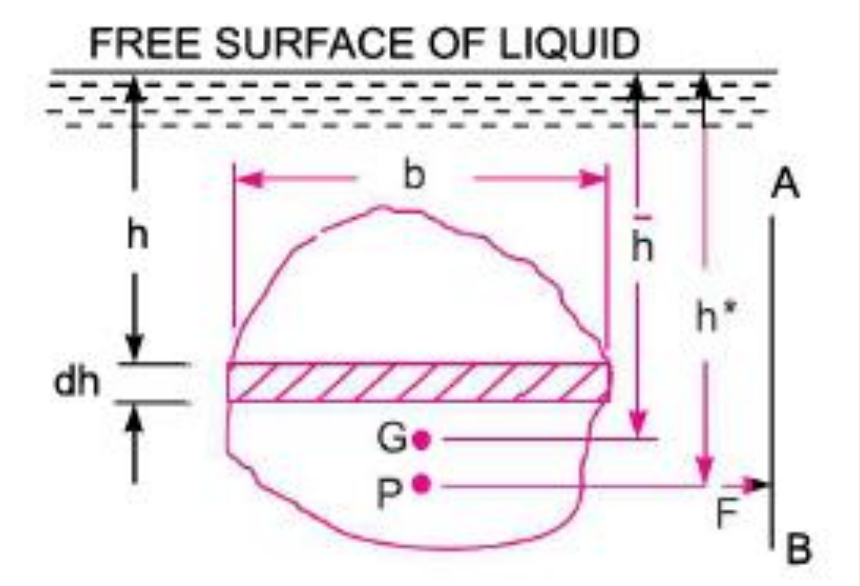
Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.



(*a*) **Total Pressure (F)**. The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

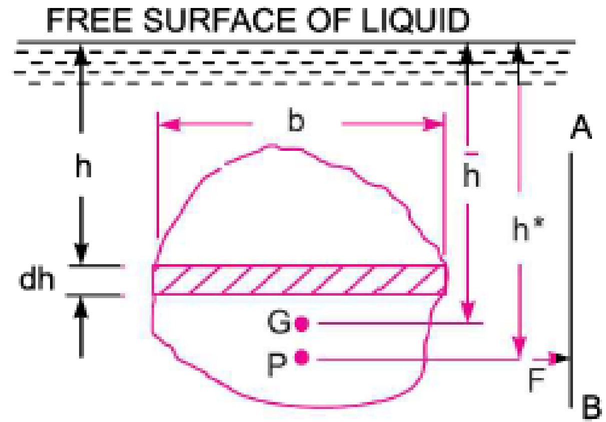


Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h} \qquad \ldots(3.1)$$

For water the value of $\rho = 1000\ kg/m^3$ and $g = 9.81\ m/s^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \qquad \{\because \quad dF = \rho g h \times b \times dh\}$$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \quad bdh = dA)$$

But
$$\int h^2 dA = \int b h^2 dh$$

= Moment of Inertia of the surface about free surface of liquid

$$= I_0$$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A\bar{h}$$

∴ $$\rho g A\bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A\bar{h}} = \frac{I_0}{A\bar{h}} \qquad ...(3.4)$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

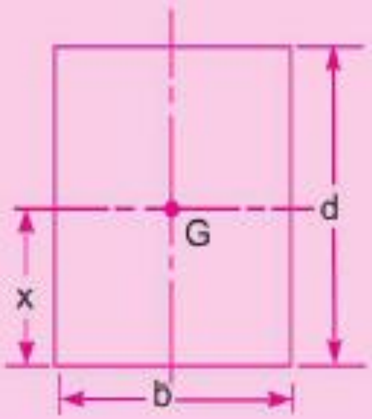
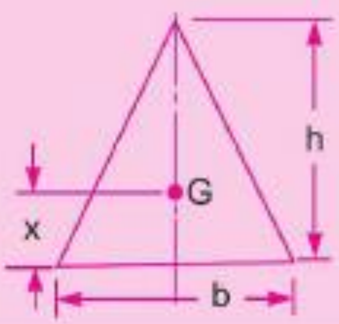
Substituting I_0 in equation (3.4), we get

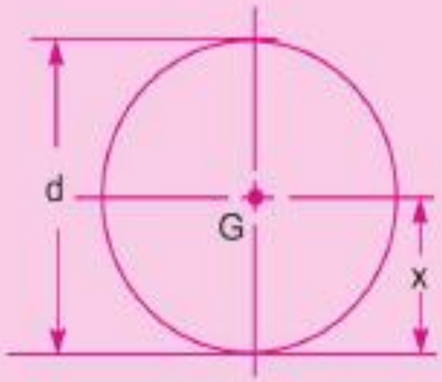
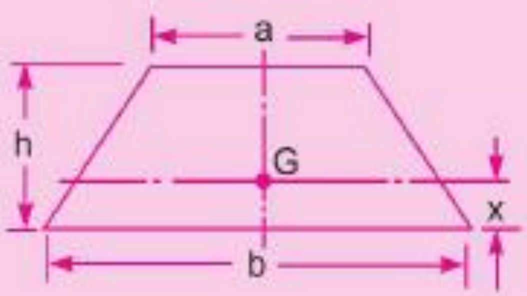
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad ...(3.5)$$

In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

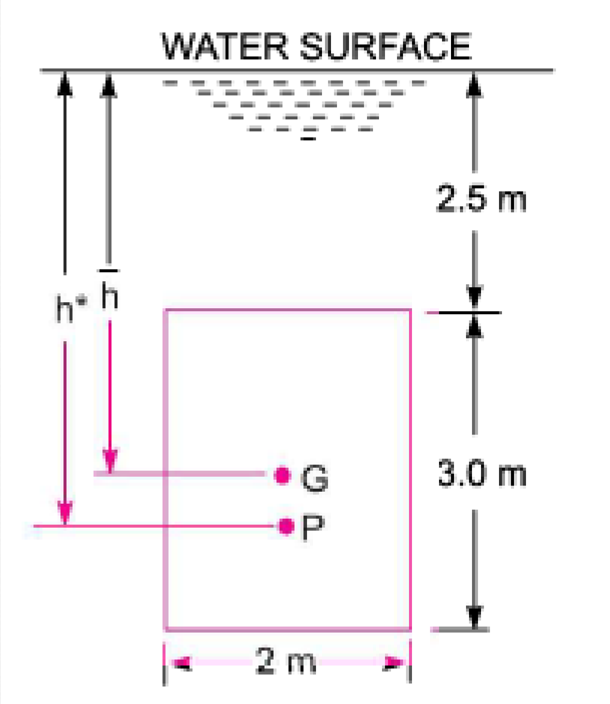
(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

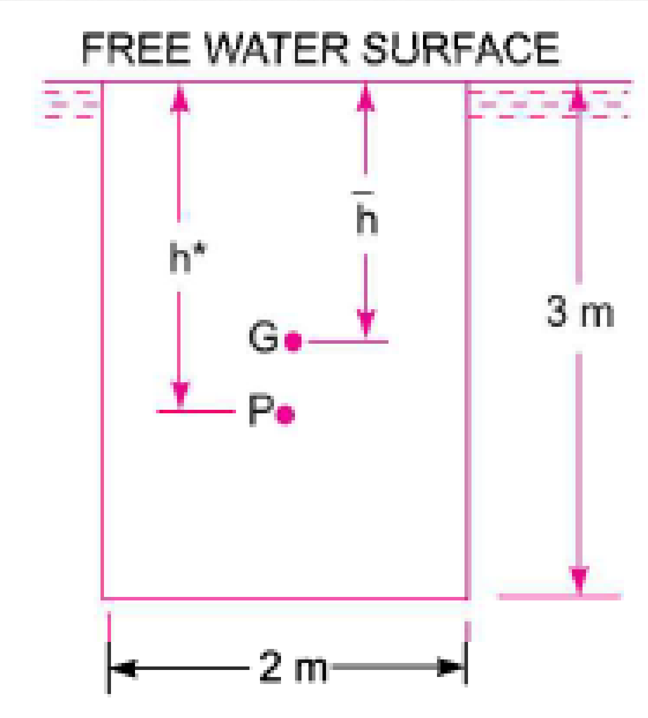
(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*





(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

$$A = 3 \times 2 = 6 \text{ m}^2, \ \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

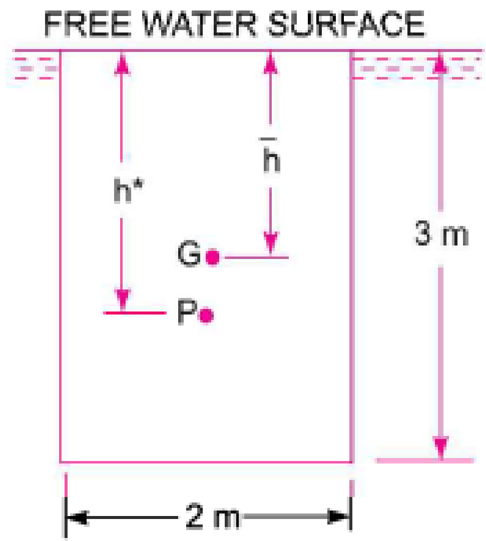


Fig. 3.2

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.\ Ans.}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440\ N.\ Ans.}$$

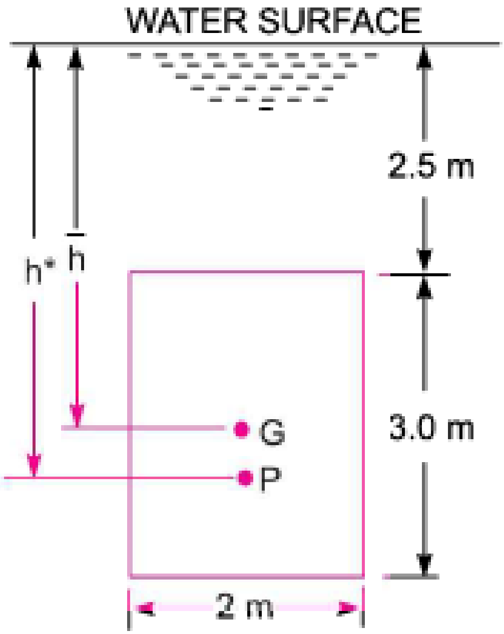


Fig. 3.3

Centre of pressure is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.\ Ans.}$$

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.*

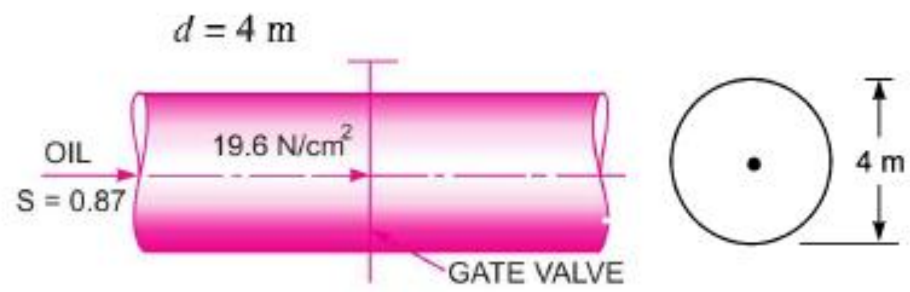


Fig. 3.7

∴ Area, $A = \frac{\pi}{4} \times 4^2 = 4\pi \text{ m}^2$

Sp. gr. of oil, $S = 0.87$

∴ Density of oil, $\rho_0 = 0.87 \times 1000 = 870 \text{ kg/m}^3$

∴ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81 \text{ N/m}^3$

Pressure at the centre of pipe, $p = 19.6 \text{ N/cm}^2 = 19.6 \times 10^4 \text{ N/m}^2$

∴ Pressure head at the centre $= \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988 \text{ m}$

∴ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \textbf{2465500 N = 2.465 MN. Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\,\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m.\ Ans.}$$

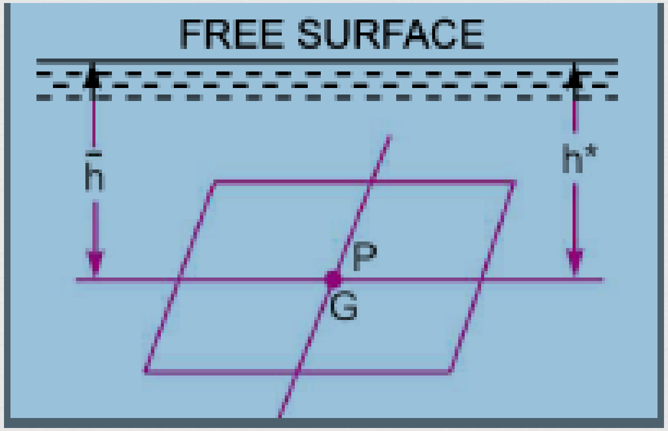
Horizontal plane surface submerged in liquid

Then total force, F, on the surface

$$= p \times \text{Area} = \rho g \times h \times A = \rho g A \bar{h}$$

where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.



▶ 3.5 INCLINED PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig. 3.18.

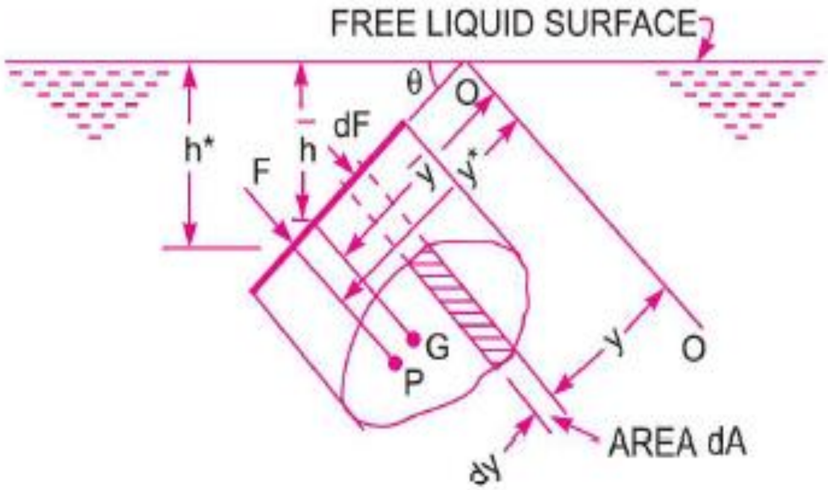


Fig. 3.18 *Inclined immersed surface.*

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h^* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho g h$

∴ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho g h \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho g h dA$

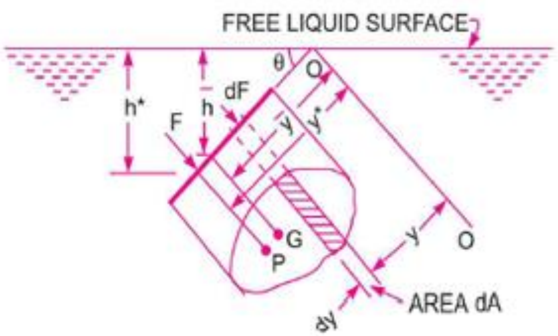


Fig. 3.18 *Inclined immersed surface.*

But from Fig. 3.18, $\dfrac{h}{y} = \dfrac{\bar{h}}{\bar{y}} = \dfrac{h^*}{y^*} = \sin\theta$

∴ $h = y \sin\theta$

∴ $F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$

But $\int y dA = A\bar{y}$

where $\bar{y}$ = Distance of C.G. from axis O-O

∴ $F = \rho g \sin\theta \, \bar{y} \times A$

$= \rho g A \bar{h}$ $\quad (\because \; \bar{h} = \bar{y}\sin\theta)$ ...(3.6)

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho g h dA$

$$= \rho g y \sin\theta \, dA \qquad [h = y \sin\theta]$$

Moment of the force, dF, about axis O-O

$$= dF \times y = \rho g y \sin\theta \, dA \times y = \rho g \sin\theta \, y^2 dA$$

Sum of moments of all such forces about O-O

$$= \int \rho g \sin\theta \, y^2 \, dA = \rho g \sin\theta \int y^2 \, dA$$

But $\int y^2 \, dA$ = M.O.I. of the surface about O-$O = I_0$

$\therefore$ Sum of moments of all forces about O-$O = \rho g \sin\theta \, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$$= F \times y^* \qquad ...(3.8)$$

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$$F \times y^* = \rho g \sin\theta \, I_0$$

or

$$y^* = \frac{\rho g \sin\theta \, I_0}{F} \qquad ...(3.9)$$

Now

$$y^* = \frac{h^*}{\sin\theta}, \; F = \rho g A \bar{h}$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Substituting these values in equation (3.9), we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\therefore \quad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

But $\quad \dfrac{\bar{h}}{\bar{y}} = \sin\theta \quad$ or $\quad \bar{y} = \dfrac{\bar{h}}{\sin\theta}$

$$\therefore \quad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \frac{\bar{h}^2}{\sin^2\theta}\right]$$

or $$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{...(3.10)}$$

If $\theta = 90°$, equation (3.10) becomes same as equation (3.5) which is applicable to vertically plane submerged surfaces.

In equation (3.10), I_G = M.O.I. of inclined surfaces about an axis passing through G and parallel to O-O.

▶ 3.6 CURVED SURFACE SUB-MERGED IN LIQUID

Consider a curved surface *AB*, sub-merged in a static fluid as shown in Fig. 3.27. Let *dA* is the area of a small strip at a depth of *h* from water surface.

Then pressure intensity on the area *dA* is $= \rho g h$

and pressure force, $$dF = p \times \text{Area} = \rho g h \times dA \qquad ...(3.11)$$

This force *dF* acts normal to the surface.

Hence total pressure force on the curved surface should be

$$F = \int \rho g h dA \qquad ...(3.12)$$

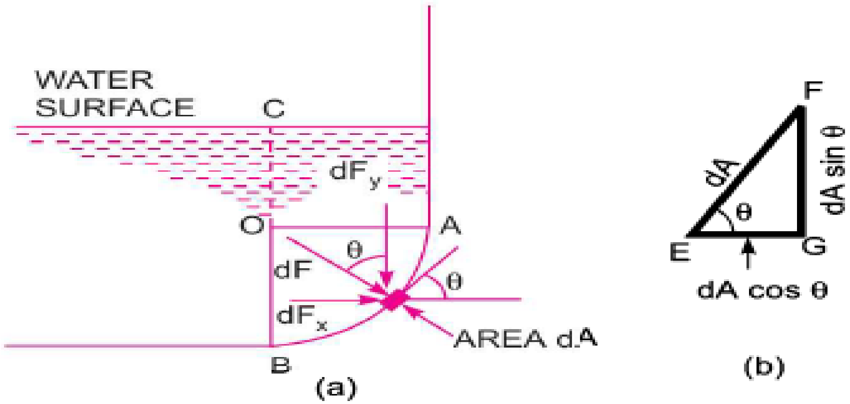


Fig. 3.27

But here as the direction of the forces on the small areas are not in the same direction, but varies from point to point. Hence integration of equation (3.11) for curved surface is impossible. The problem can, however, be solved by resolving the force dF in two components dF_x and dF_y in the x and y directions respectively. The total force in the x and y directions, *i.e.*, F_x and F_y are obtained by integrating dF_x and dF_y. Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2} \qquad ...(3.13)$$

and inclination of resultant with horizontal is $\tan \phi = \frac{F_y}{F_x}$ ...(3.14)

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin \theta = \rho g h dA \sin \theta \qquad \{\because dF = \rho g h dA\}$$

and

$$dF_y = dF \cos \theta = \rho g h dA \cos \theta$$

Total forces in the x and y direction are :

$$F_x = \int dF_x = \int \rho g h dA \sin \theta = \rho g \int h dA \sin \theta \qquad ...(3.15)$$

and

$$F_y = \int dF_y = \int \rho g h dA \cos \theta = \rho g \int h dA \cos \theta \qquad ...(3.16)$$

Fig. 3.27 (*b*) shows the enlarged area dA. From this figure, *i.e.*, ΔEFG,

$$EF = dA$$
$$FG = dA \sin \theta$$
$$EG = dA \cos \theta$$

Thus in equation (3.15), $dA \sin \theta = FG$ = Vertical projection of the area dA and hence the expression $\rho g \int h dA \sin \theta$ represents the total pressure force on the projected area of the curved surface on the vertical plane. Thus

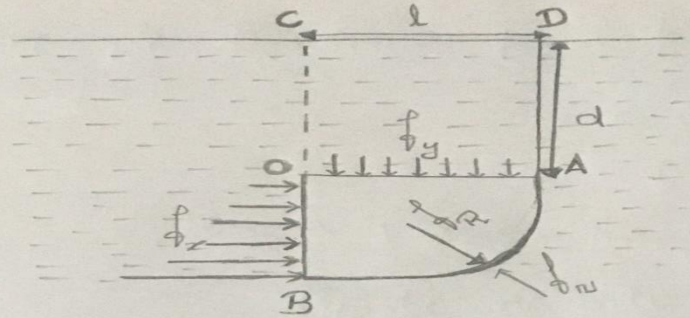
F_x = Total pressure force on the projected area of the curved surface on vertical plane. ...(3.17)

Also $dA \cos \theta = EG$ = horizontal projection of dA and hence $h dA \cos \theta$ is the volume of the liquid contained in the elementary area dA upto free surface of the liquid. Thus $\int h dA \cos \theta$ is the total volume contained between the curved surface extended upto free surface.

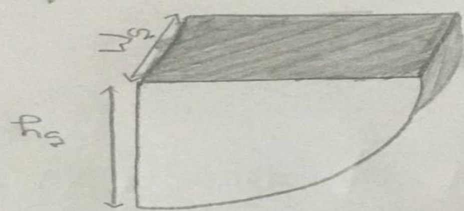
Hence $\rho g \int h dA \cos \theta$ is the total weight supported by the curved surface. Thus

$$F_y = \rho g \int h dA \cos \theta$$

= weight of liquid supported by the curved surface upto free surface of liquid. ...(3.18)



Curved surface submerged in liquid



Total force acting in the curved surface

$$F_R = \sqrt{F_x^2 + F_y^2}$$

F_x = Total Pressure force acting on the Projected area.

F_y = Weight of the liquid supported by the curved surface.

$F_x = \rho g \bar{h} A$ → A → is surface area in the vertical plan

F_y = Weight of liquid supported by the curved surface.

$$W = m \times g$$

$$\rho = m/V \text{ , so } m = \rho . V$$

Sub mass value in weight.

$$W = \rho . g . V$$

here V - volume.

from the figure.

F_y = Weight of DAOC + Weight of water in AOB

$$= \rho . g \, V_1 (\text{of DAOC}) + \rho . g . V_2 (\text{of AOB})$$

$$F_y = \rho . g \left(l \times w \times h + \frac{\pi}{4} r^2 \times b \right)$$

Problem 3.14 (*a*) *A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

(*i*) **Total pressure** force is given by equation (3.6) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$

$A = b \times d = 3 \times 2 = 6 \text{ m}^2$

∴ $\bar{h}$ = Depth of C.G. from free water surface

= 1.5 + 1.5 sin 30°

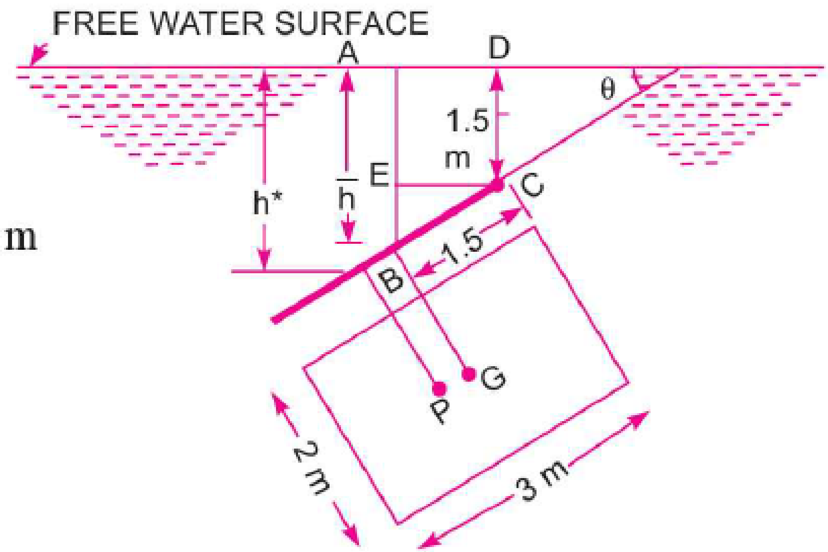


Fig. 3.19

$\{\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

∴ $F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$h^* = \frac{4.5 \times \sin^2 30^\circ}{6 \times 2.25} + 2.25 = \frac{4.5 \times \frac{1}{4}}{6 \times 2.25} + 2.25$$

$$= 0.0833 + 2.25 = \mathbf{2.3333 \text{ m. Ans.}}$$

Problem 3.21 *Find the total pressure and position of centre of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.*

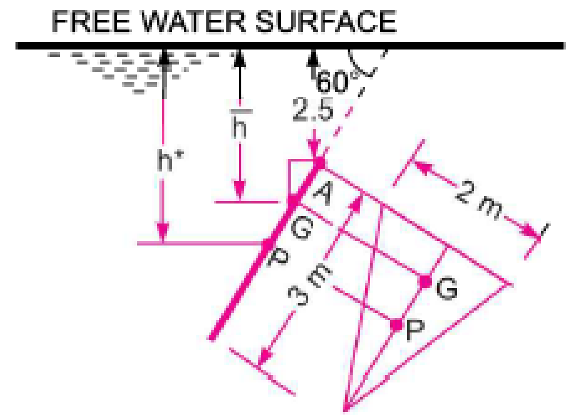


Fig. 3.26

Solution. Given :

Base of plate, $b = 2$ m

Height of plate, $h = 3$ m

$\therefore$ Area, $A = \dfrac{b \times h}{2} = \dfrac{2 \times 3}{2} = 3 \text{ m}^2$

Inclination, $\theta = 60°$

Depth of centre of gravity from free surface of water,

$$\bar{h} = 2.5 + AG \sin 60°$$

$$= 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2} \quad \left\{\because AG = \frac{1}{3} \text{ of height of triangle}\right\}$$

$$= 2.5 + .866 \text{ m} = 3.366 \text{ m}$$

(*i*) **Total pressure force (F)**

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3 \times 3.366 = \mathbf{99061.38 \text{ N. Ans.}}$$

(*ii*) **Centre of pressure (h*).** Depth of centre of pressure from free surface of water is given by

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $I_G = \dfrac{bh^3}{36} = \dfrac{2 \times 3^3}{36} = \dfrac{3}{2} = 1.5 \text{ m}^4$

$$\therefore \quad h^* = \frac{1.5 \times \sin^2 60°}{3 \times 3.366} + 3.366 = 0.111 + 3.366 = \mathbf{3.477 \text{ m. Ans.}}$$

Problem 3.23 *Fig. 3.30 shows a gate having a quadrant shape of radius 2 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.*

Solution. Given :

Radius of gate = 2 m

Width of gate = 1 m

Horizontal Force

F_x = Force on the projected area of the curved surface on vertical plane

= Force on $BO = \rho g A \bar{h}$

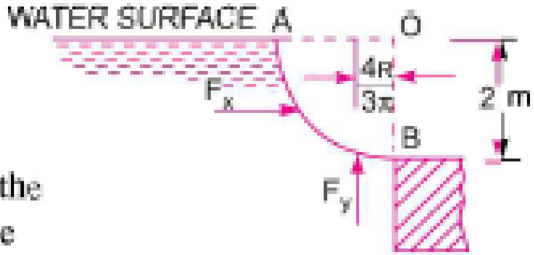


Fig. 3.30

where A = Area of $BO = 2 \times 1 = 2\ m^2$, $\bar{h} = \frac{1}{2} \times 2 = 1$ m ;

$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620$ N

Problem 3.24 *Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4.0 m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.*

Problem 3.25 *Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90° sector of radius 4 m as shown in Fig. 3.32. Take width of gate unity.*

Solution. Given :

Radius of gate, $R = 4$ m

Horizontal component of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$= \rho g A \bar{h}$

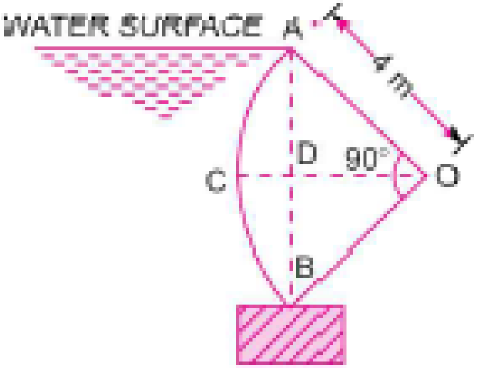


Fig. 3.32

where $A = AB \times$ Width of gate

$= 2 \times AD \times 1$ $(\because AB = 2AD)$

$= 2 \times 4 \times \sin 45° = 8 \times .707 = 5.656 \text{ m}^2$ $\{\because AD = 4 \sin 45°\}$

$$\bar{h} = \frac{AB}{2} = \frac{5.656}{2} = 2.828 \text{ m}$$

$\therefore$ $F_x = 1000 \times 9.81 \times 5.656 \times 2.828 \text{ N} =$ **156911 N. Ans.**

Buoyancy force

- Buoyancy or up thrust, is an upward force exerted by a fluid that opposes the weight of a partially or fully imm... ...ncreases wit... ...g fluid.

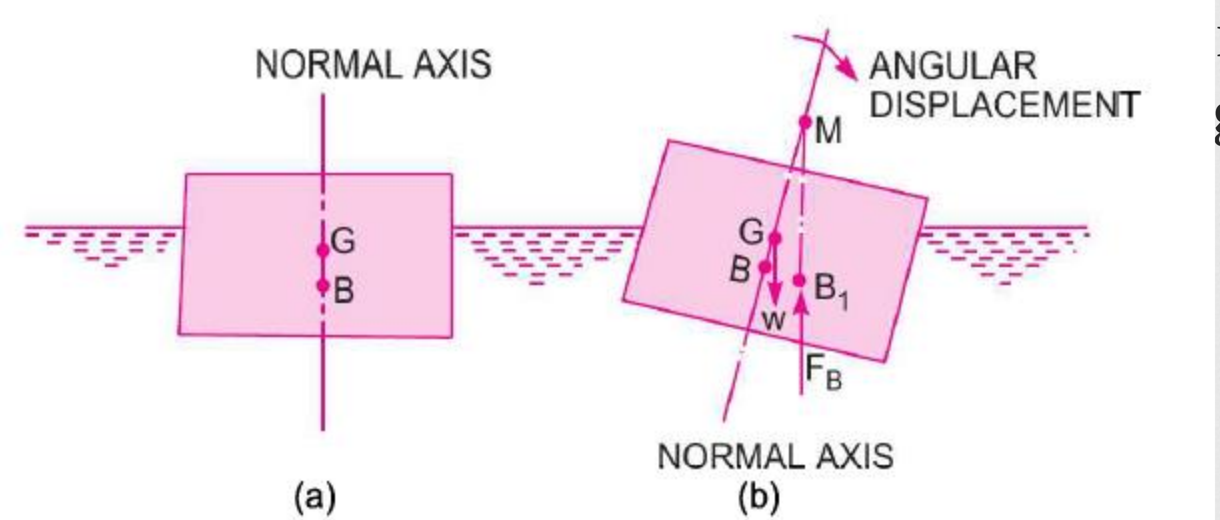


Fig. 4.5 ***Meta-centre***

Problem 4.1 *Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length 6.0 m.*

Solution. Given :

Width $= 2.5$ m

Depth $= 1.5$ m

Length $= 6.0$ m

Volume of the block $= 2.5 \times 1.5 \times 6.0 = 22.50 \text{ m}^3$

Density of wood, $\rho = 650 \text{ kg/m}^3$

$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$

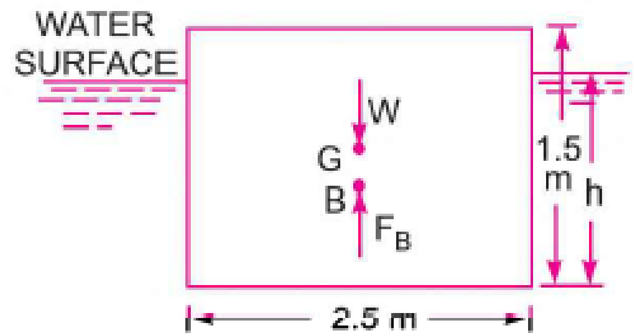


Fig. 4.1

For equilibrium the weight of water displaced = Weight of wooden block

$= 143471$ N

$\therefore$ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625 \text{ m}^3}. \text{ **Ans.**}$$

($\because$ Weight density of water $= 1000 \times 9.81 \text{ N/m}^3$)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

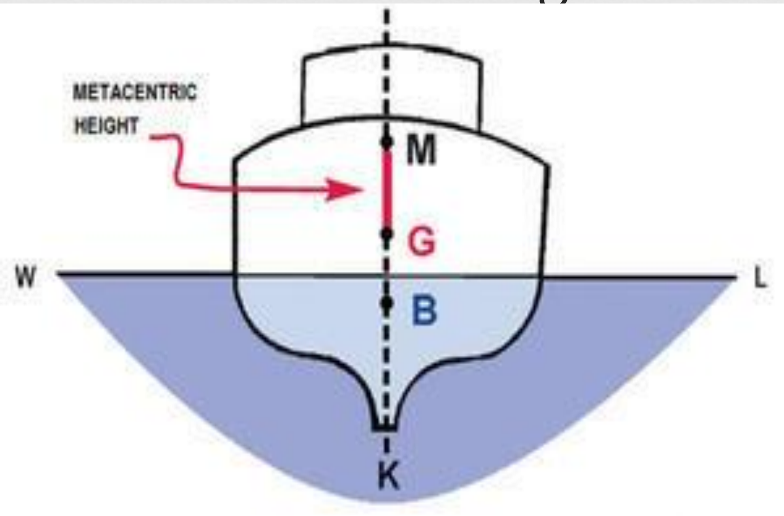
r $2.5 \times h \times 6.0 = 14.625 \text{ m}^3$, where h is depth of wooden block in water

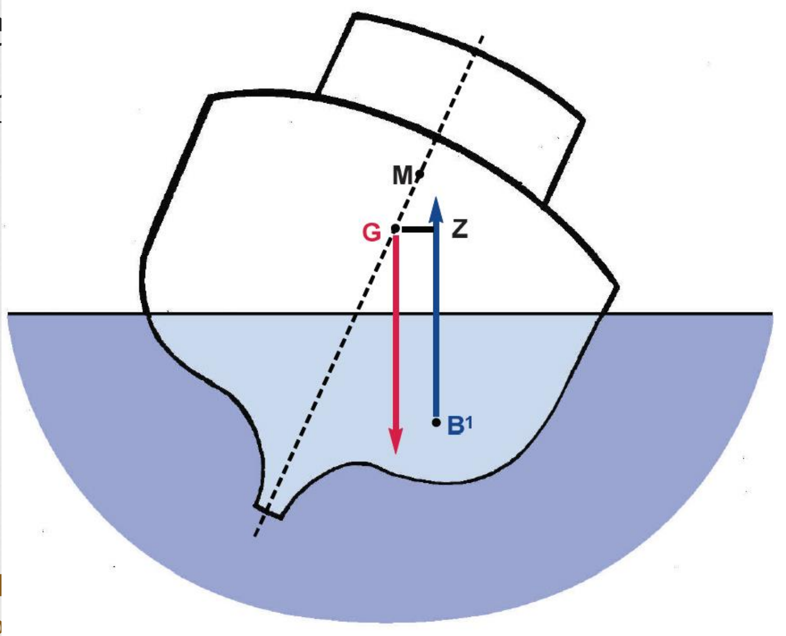
$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875 \text{ m from base.}} \text{ **Ans.**}$$

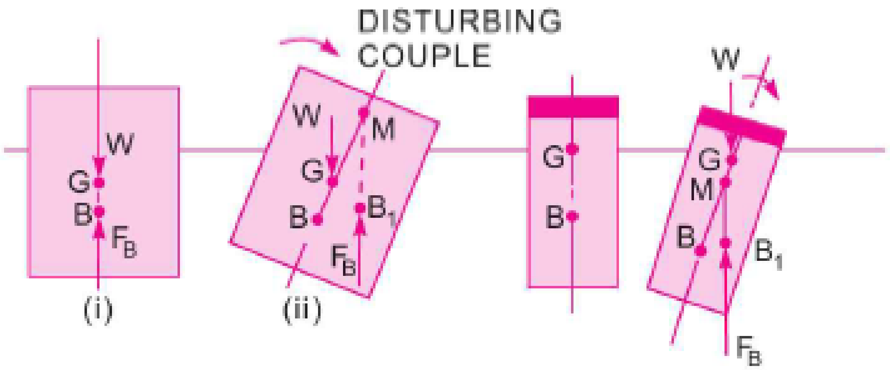
Metacentric height

- The metacentric height is a measurement of the initial static stability of a floating body. It is calculated as the distance between the center of gravity of a ship and its metacenter. A larger metace
ur





(*a*) **Stable Equilibrium.** If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(a) Stable equilibrium M is above G (b) Unstable equilibrium M is below G.

Fig. 4.13 *Stability of floating bodies.*

(*b*) **Unstable Equilibrium.** If the point M is below G, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Metacentric height

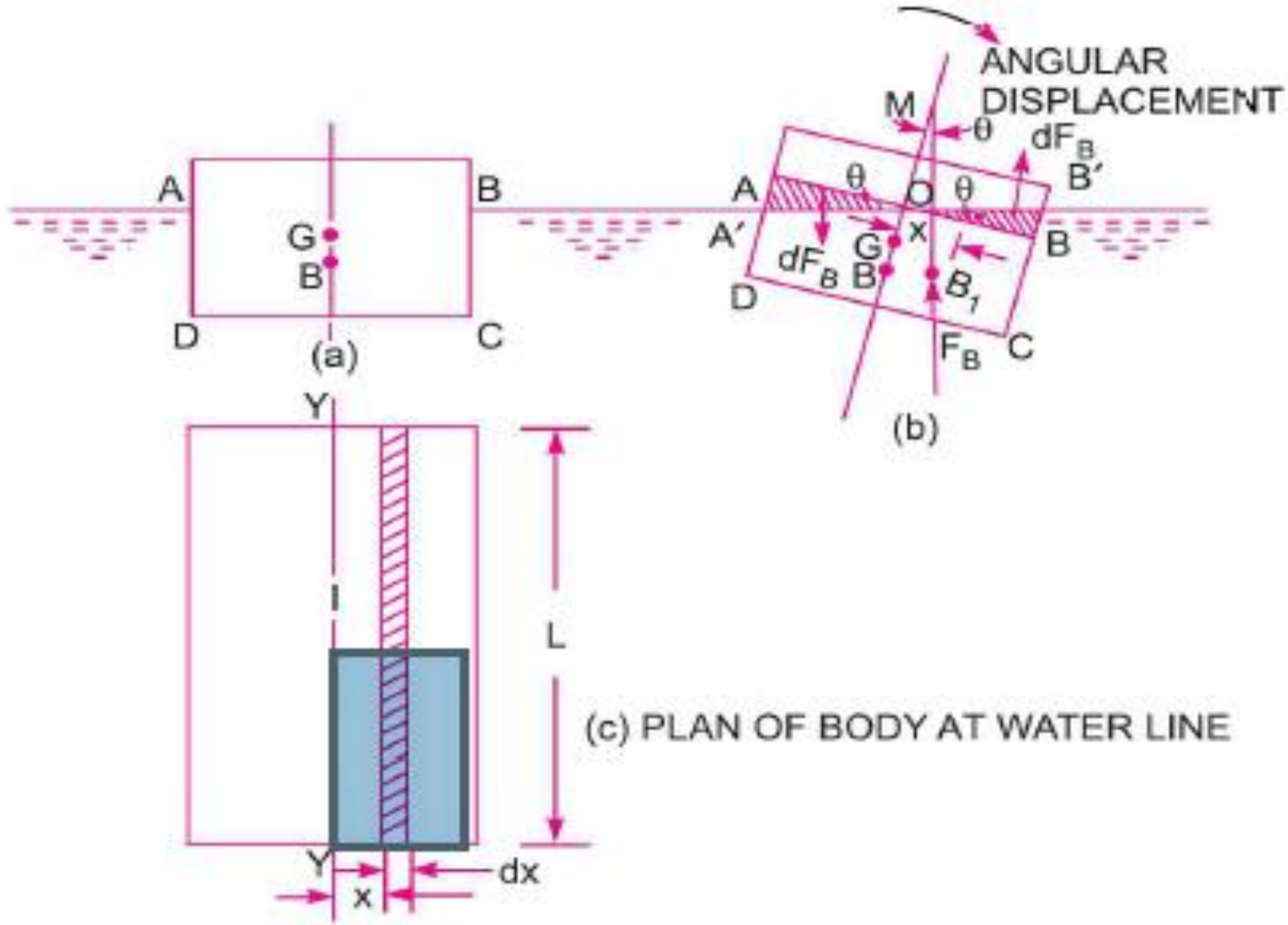


Fig. 4.6 *Meta-centre height of floating body.*

Pressure distribution in a liquid due to constant acceleration

The following are the important cases under consideration :

(*i*) Liquid containers subject to constant horizontal acceleration.

(*ii*) Liquid containers subject to constant vertical acceleration.

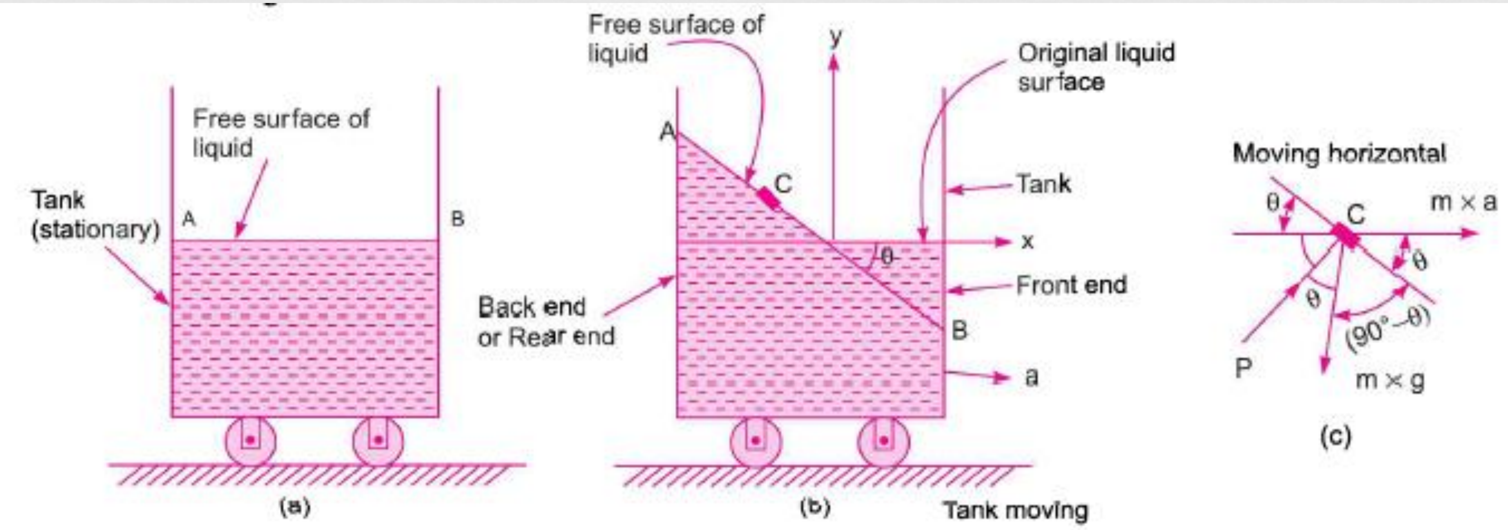


Fig. 3.43

3.8.1 Liquid Containers Subject to Constant Horizontal Acceleration. Fig. 3.43 (*a*) shows a tank containing a liquid upto a certain depth. The tank is stationary and free surface of liquid is horizontal. Let this tank is moving with a constant acceleration '*a*' in the horizontal direction towards right as shown in Fig. 3.43 (*b*). The initial free surface of liquid which was horizontal, now takes the shape as shown in Fig. 3.43 (*b*). Now *AB* represents the new free surface of the liquid. Thus the free surface of liquid due to horizontal acceleration will become a downward sloping inclined plane, with the liquid rising at the back end, the liquid falling at the front end. The equation for the free liquid surface can be derived by considering the equilibrium of a fluid element C lying on the free surface. The forces acting on the element *C* are :

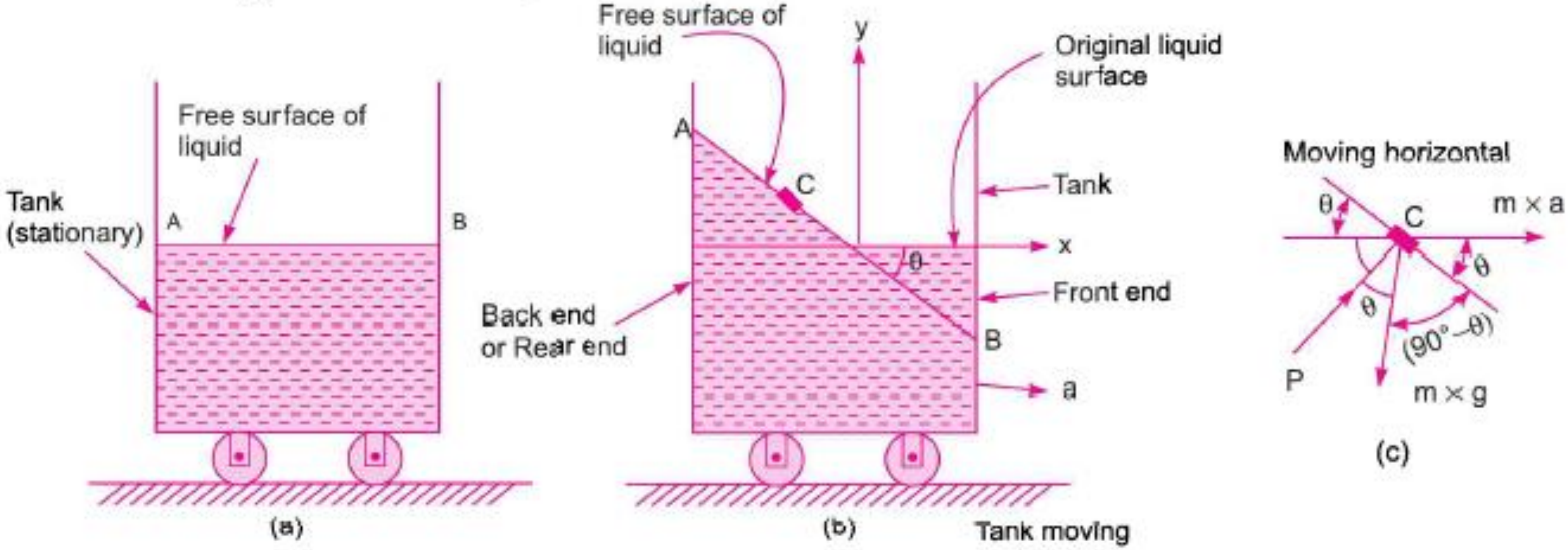


Fig. 3.43

(*i*) the pressure force *P* exerted by the surrounding fluid on the element *C*. This force is normal to the free surface.

(*ii*) the weight of the fluid element *i.e.*, $m \times g$ acting vertically downward.

(*iii*) accelerating force *i.e.*, $m \times a$ acting in horizontal direction.

Problem 3.34 *A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of 2.4 m/s². The length, width and depth of the tank are 6 m, 2.5 m and 2 m respectively. If the depth of water in the tank is 1 m and tank is open at the top then calculate :*

(*i*) *the angle of the water surface to the horizontal,*

(*ii*) *the maximum and minimum pressure intensities at the bottom,*

(*iii*) *the total force due to water acting on each end of the tank.*

Solution. Given :

Constant acceleration, $a = 2.4$ m/s².

Length = 6 m ; Width = 2.5 m and depth = 2 m.

Depth of water in tank, $h = 1$ m

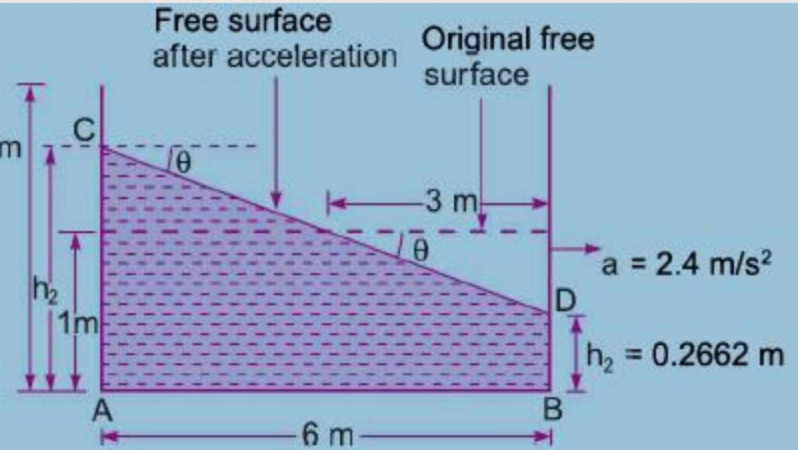


Fig. 3.45

(*i*) **The angle of the water surface to the horizontal**

Let θ = the angle of water surface to the horizontal

Using equation (3.20), we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig. 3.45)

$\therefore$ $\tan\theta = 0.2446$ (slope downward)

$\therefore$ $\theta = \tan^{-1} 0.2446 =$ **13.7446°** or **13° 44.6′. Ans.**

(*ii*) **The maximum and minimum pressure intensities at the bottom of the tank**

From the Fig. 3.45,

Depth of water at the front end,

$$h_1 = 1 - 3 \tan \theta = 1 - 3 \times 0.2446 = 0.2662 \text{ m}$$

Depth of water at the rear end,

$$h_2 = 1 + 3 \tan \theta = 1 + 3 \times 0.2446 = 1.7338 \text{ m}$$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now the maximum pressure intensity at the bottom will be at point *A* and it is given by,

$$p_{max} = \rho \times g \times h_2$$

$$= 1000 \times 9.81 \times 1.7338 \text{ N/m}^2 = \mathbf{17008.5 \text{ N/m}^2. \text{ Ans.}}$$

The minimum pressure intensity at the bottom will be at point B and it is given by

$$p_{min} = \rho \times g \times h_1$$

$$= 1000 \times 9.81 \times 0.2662 = \mathbf{2611.4 \text{ N/m}^2. \text{ Ans.}}$$

(*iii*) **The total force due to water acting on each end of the tank**

Let F_1 = total force acting on the front side (*i.e.*, on face *BD*)

F_2 = total force acting on the rear side (*i.e.*, on face *AC*)

Then $F_1 = \rho g A_1 \bar{h}_1$, where $A_1 = BD \times \text{width of tank} = h_1 \times 2.5 = 0.2662 \times 2.5$

and

$$\bar{h}_1 = \frac{BD}{2} = \frac{h_1}{2} = \frac{0.2662}{2} = 0.1331 \text{ m}$$

$$= 1000 \times 9.81 \times (0.2662 \times 2.5) \times 0.1331$$

$$= \mathbf{868.95 \text{ N. Ans.}}$$

and $F_2 = \rho.g.A_2.\bar{h}_2$, where $A_2 = AB \times$ width of tank $= h_2 \times 2.5 = 1.7338 \times 2.5$

$$\bar{h}_2 = \frac{AB}{2} = \frac{h_2}{2} = \frac{1.7338}{2} = 0.8669 \text{ m}$$

$$= 1000 \times 9.81 \times (1.7338 \times 2.5) \times 0.8669$$

$$= \mathbf{36861.8\ N.\ Ans.}$$

$\therefore$ Resultant force $= F_1 - F_2$

$$= 36861.8 \text{ N} - 868.95$$

$$= 35992.88 \text{ N}$$

Note. The difference of the forces acting on the two ends of the tank is equal to the force necessary to accelerate the liquid mass. This can be proved as shown below :

Consider the control volume of the liquid *i.e.*, control volume is *ACDBA* as shown in Fig. 3.46. The net force acting on the control volume in the horizontal direction must be equal to the product of mass of the liquid in control volume and acceleration of the liquid.

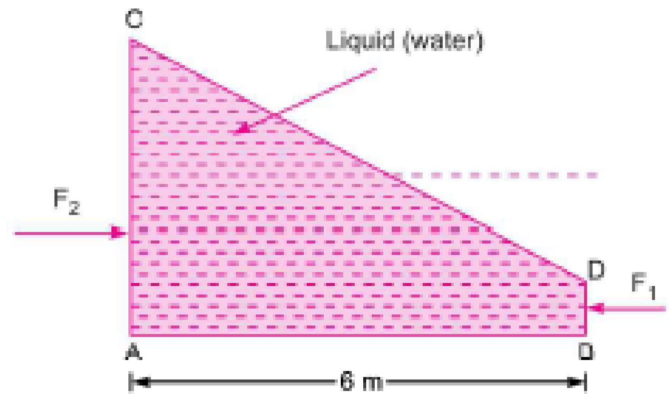


Fig. 3.46

$\therefore$ $(F_1 - F_2) = M \times a$

$$= (\rho \times \text{volume of control volume}) \times a$$

$$= (1000 \times \text{Area of } ABDCE \times \text{width}) \times 2.4$$

$$= \left[1000 \times \left(\frac{AC + BD}{2}\right) \times AB \times \text{width}\right] \times 2.4$$

$$\left[\because \text{ Area of trapezium} = \left(\frac{AC + BD}{2}\right) \times AB\right]$$

$$= 1000 \times \left(\frac{1.7338 + 0.2662}{2}\right) \times 6 \times 2.5 \times 2.4$$

$$= 36000 \text{ N}$$

($\because AC = h_2 = 1.7338$ m, $BD = h_1 = 0.2662$ m, and $AB = 6$ m, width $= 2.5$ m)

The above force is nearly the same as the difference of the forces acting on the two ends of the tank. (*i.e.*, $35992.88 \simeq 36000$).

Problem 3.35 *The rectangular tank of the above problem contains water to a depth of 1.5 m. Find the horizontal acceleration which may be imparted to the tank in the direction of its length so that*

(*i*) *the spilling of water from the tank is just on the verge of taking place,*

(*ii*) *the front bottom corner of the tank is just exposed,*

(*iii*) *the bottom of the tank is exposed upto its mid-point.*

Also calculate the total forces exerted by the water on each end of the tank in each case. Also prove that the difference between these forces is equal to the force necessary to accelerate the mass of water tank.

Solution. Given :

Dimensions of the tank from previous problem,

$$L = 6 \text{ m, width } (b) = 2.5 \text{ m and depth} = 2 \text{ m}$$